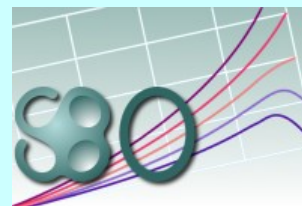
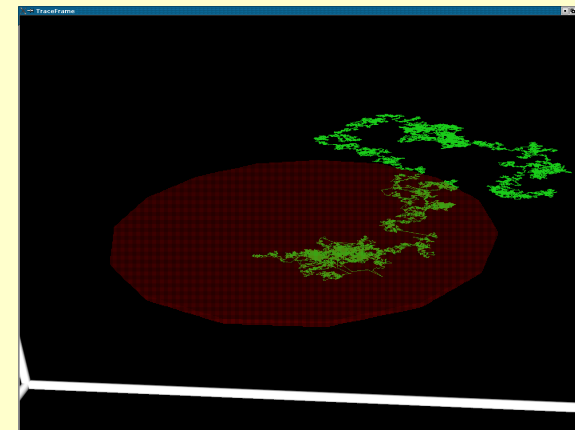
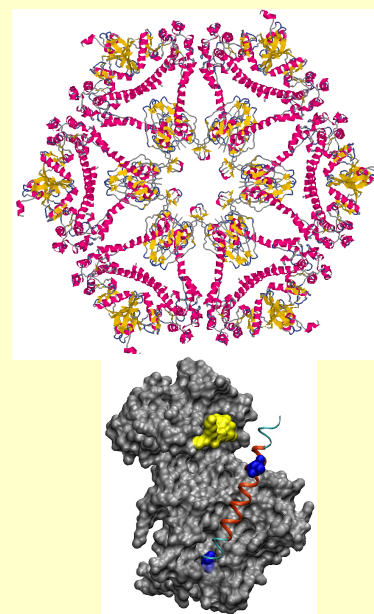
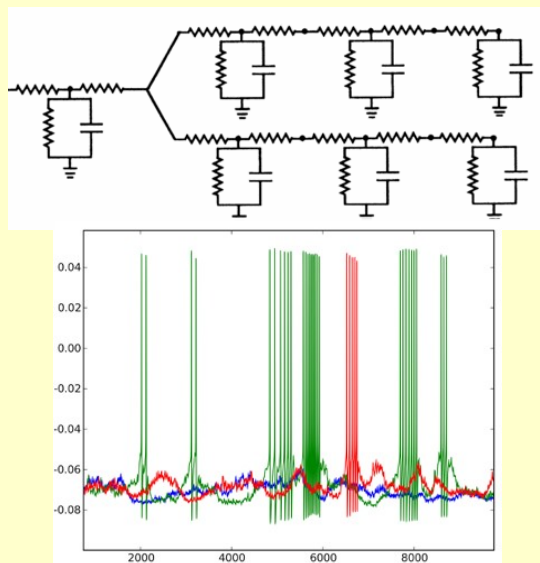
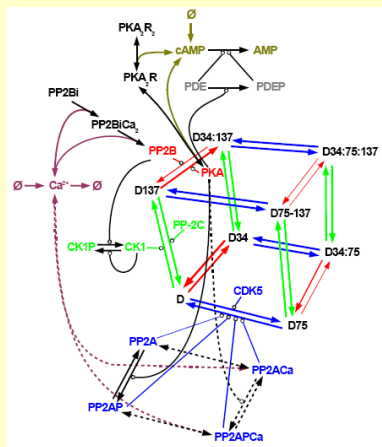


Computational Neurobiology



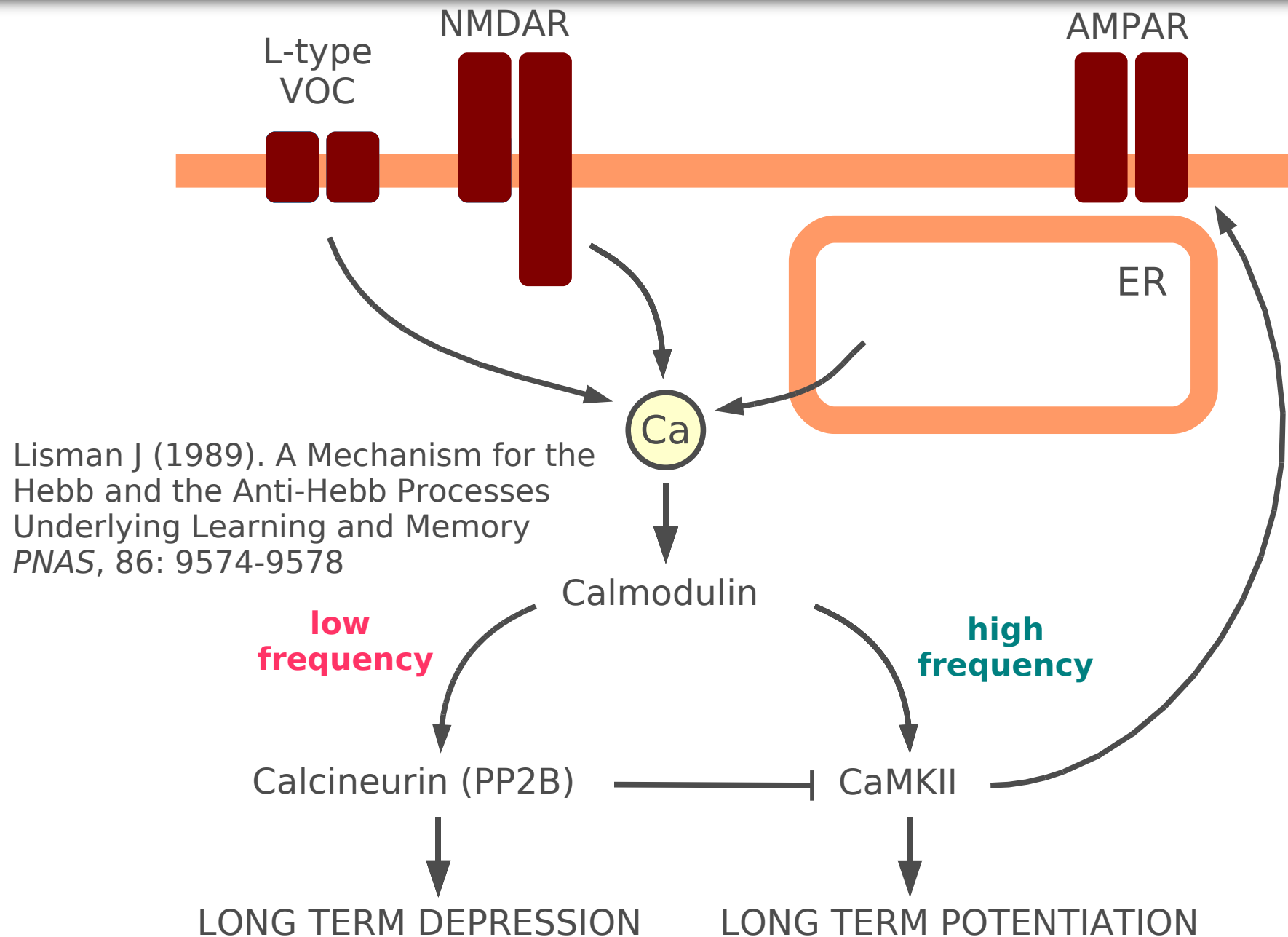
Systems Biology

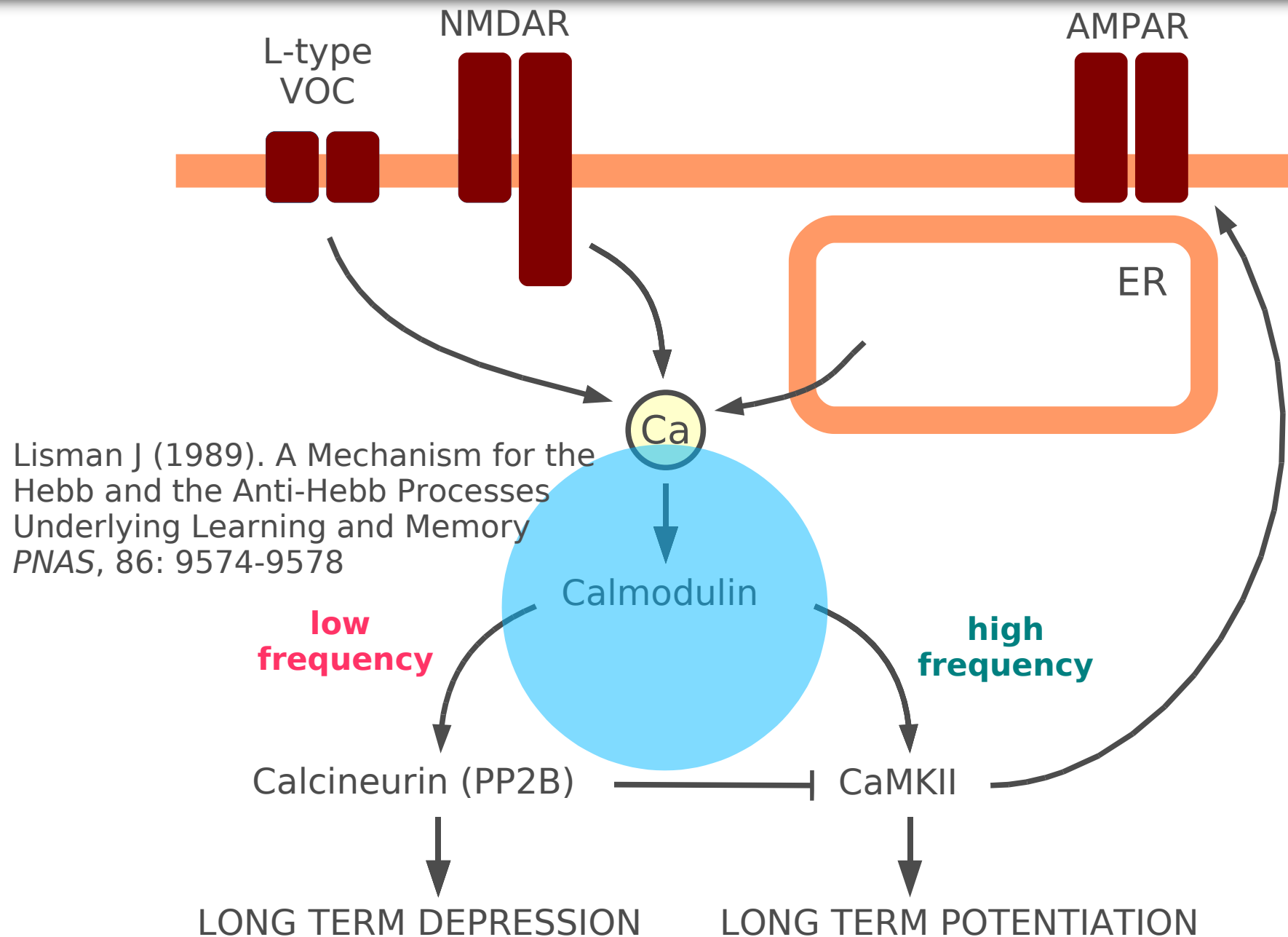


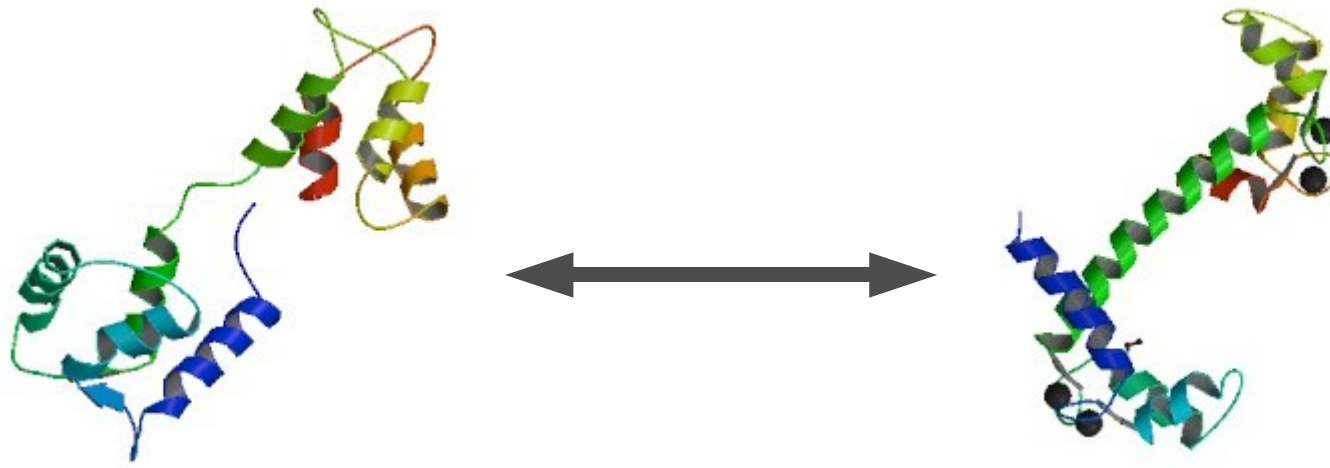
Cooperativity in signal transduction

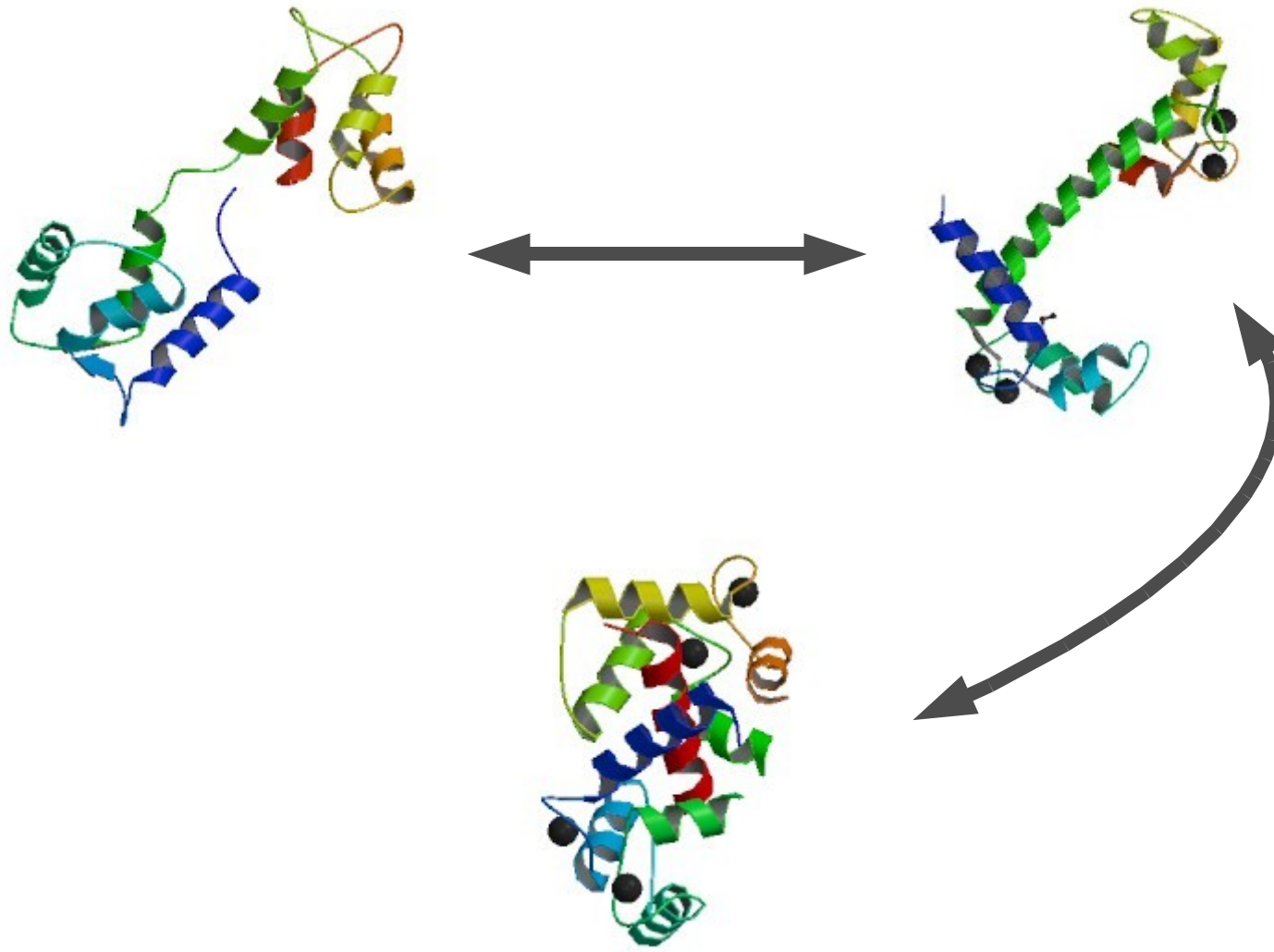
Nicolas Le Novère

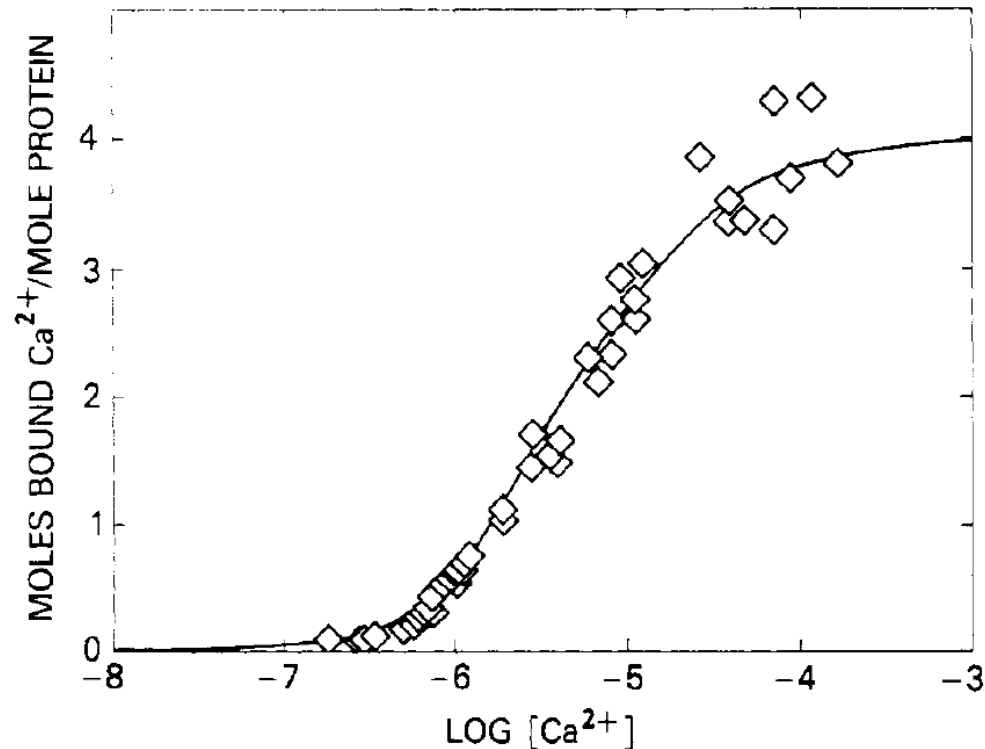




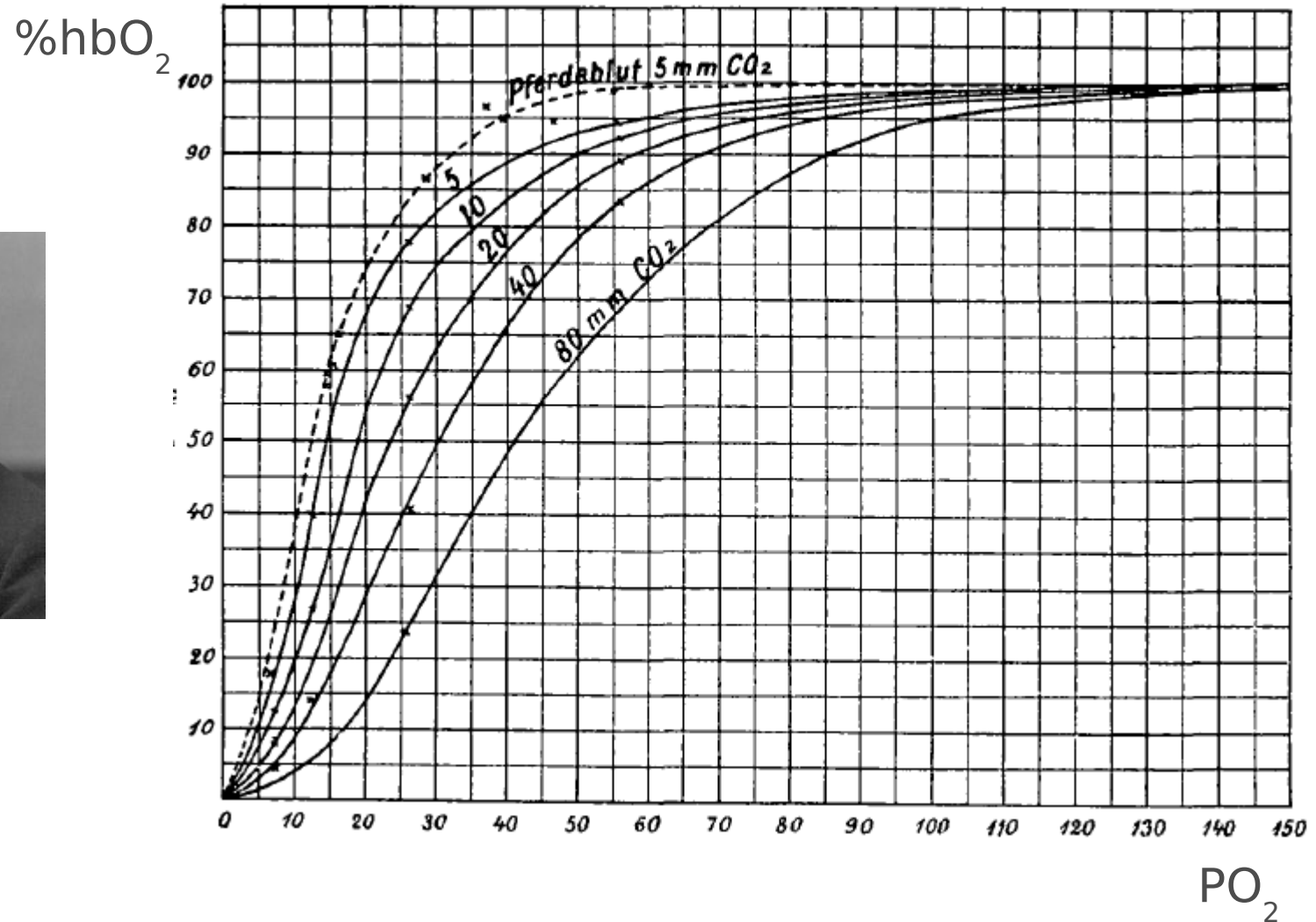








Crouch TH and Klee CB (1980)
Positive cooperative binding of calcium to bovine brain calmodulin.
Biochemistry, 19: 3692-3698



Bohr C (1903) Theoretische behandlung der quantitativen verhältnisse bei der sauerstoff aufnahme des hämoglobins *Zentralbl Physiol* 17: 682



iv PROCEEDINGS OF THE PHYSIOLOGICAL

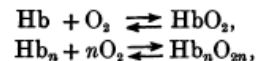
The possible effects of the aggregation of the molecules of hæmoglobin on its dissociation curves. By A. V. HILL.

In a previous communication Barcroft and I gave evidence which seemed to us to prove conclusively that dialysed hæmoglobin consists simply of molecules containing each one atom of iron. The molecular weight is therefore $Hb = 16,660$. These experiments have not been published yet, but I shall assume the results.

Other observers (Reid, Roaf, Hüfner and Gansser) working on different solutions have obtained divergent results. The method used by all of them was the direct estimation of the osmotic pressure, by means of a membrane permeable to salts, but not to hæmoglobin. The method involves a relatively large error, because the quantity measured is small. It is doubtful however whether this can explain the discordant results.

Our work led me to believe that the divergence between the results of different observers was due to an aggregation of the hæmoglobin molecules by the salts present in the solution, a consequent lowering of the number of molecules, and an increase in the average molecular weight as observed by the osmotic pressure method. To test this hypothesis I have applied it to several of the dissociation curves obtained by Barcroft and Camis with hæmoglobin in solutions of various salts, and with hæmoglobin prepared by Bohr's method.

The equation for the reaction would be



where Hb_n represents the aggregate of n molecules of Hb . I have supposed that in every solution there are many different sized aggregates, corresponding to many values of n .

If there were in the solution only Hb and Hb_2 the dissociation curve would be

$$y = \lambda \frac{K'x^2}{1 + K'x^2} + (100 - \lambda) \frac{Kx}{1 + Kx} \dots\dots\dots (A),$$

where $\lambda\%$ is as Hb_2 , $(100 - \lambda)\%$ as Hb , K' is the equilibrium constant of the reaction $Hb_2 + 2O_2 \rightleftharpoons Hb_2O_4$ and K that of $Hb + O_2 \rightleftharpoons HbO_2$: K has the value '125 (Barcroft and Roberts).

Hill AV (1910) The possible effects of the aggregation of the molecules of hæmoglobin on its dissociation curves. *J Physiol* 40: iv-vii.



iv PROCEEDINGS OF THE PHYSIOLOGICAL

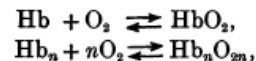
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Now it is unlikely that in either of these cases there is only Hb and Hb_2 : and as the calculation of the constants in these equations is very tedious I decided to try whether the equation

$$y = 100 \frac{Kx^n}{1 + Kx^n} \dots\dots\dots(B)$$

would satisfy the observations.



iv PROCEEDINGS OF THE PHYSIOLOGICAL

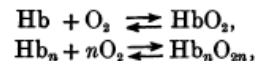
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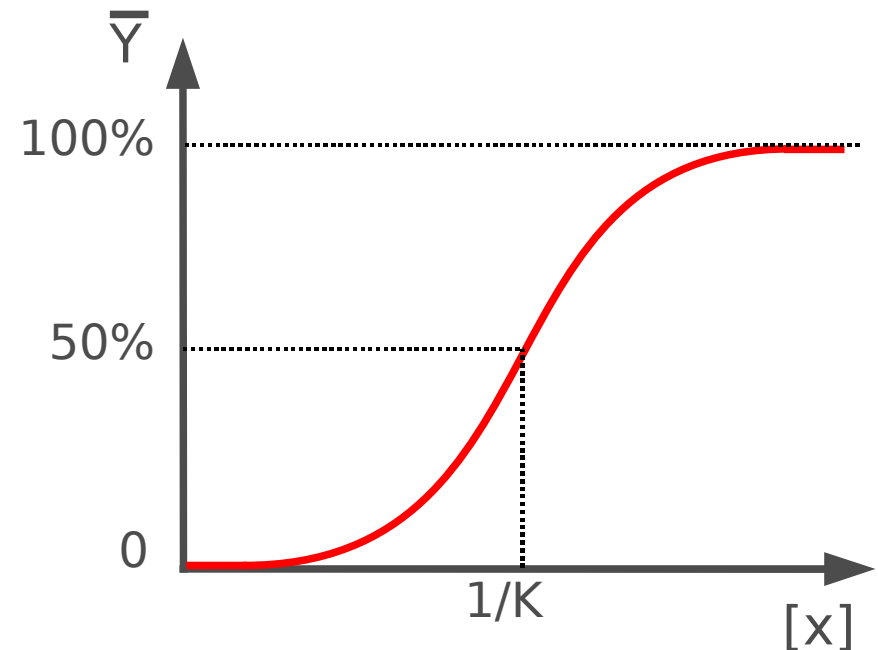
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$$\bar{Y} = \frac{K^n [x]^n}{1 + K^n [x]^n}$$

Hill equation

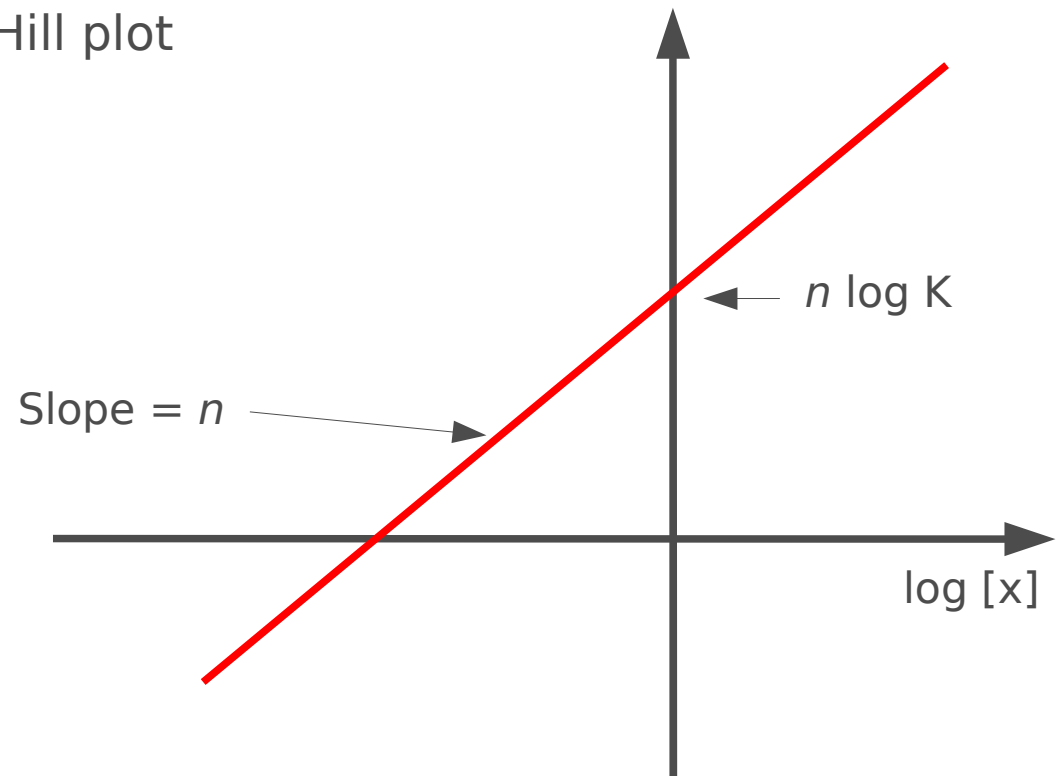


$$\bar{Y} = \frac{K^n [x]^n}{1 + K^n [x]^n}$$

Hill equation

$$\log \frac{\bar{Y}}{1 - \bar{Y}} = n \log [x] + n \log K$$

Hill plot



$$\bar{Y} = \frac{K^n [x]^n}{1 + K^n [x]^n}$$

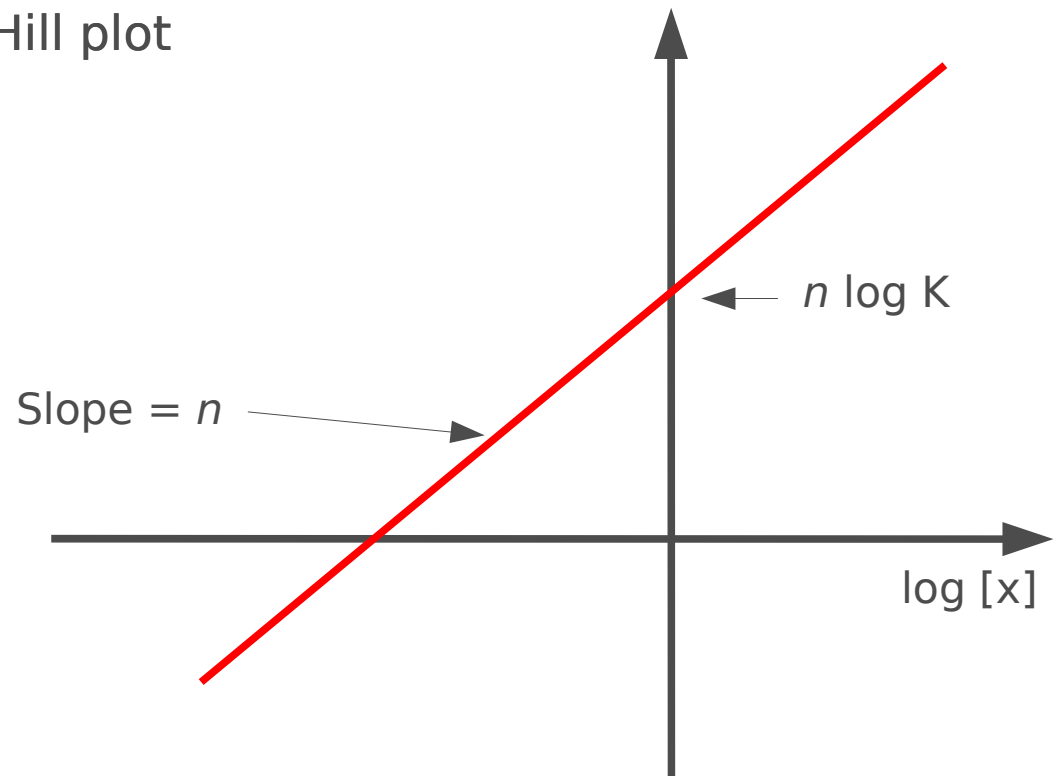
Hill equation

$$\log \frac{\bar{Y}}{1 - \bar{Y}} = n \log [x] + n \log K$$

Hill plot

Effect increases in function of
the signal to the power of n :
 $n > 1$, ultra-sensitive
 $n < 1$, infra-sensitive

BUT cooperativity of ligand,
not of binding sites: unique affinity



THE HEMOGLOBIN SYSTEM.

VI. THE OXYGEN DISSOCIATION CURVE OF HEMOGLOBIN

By G. S. ADAIR.

WITH THE COLLABORATION OF A. V. BOCK AND H. FIELD, JR.
 (From the Medical Laboratories of the Massachusetts General Hospital,
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(Received for publication, January 7, 1925.)

This work gives the oxygen dissociation curves of so
previously investigated in regard to their acid-binding and

Adair GS (1925) The hemoglobin system
 VI. the oxygen dissociation curve of
 hemoglobin. *J Biol Chem* 63: 529

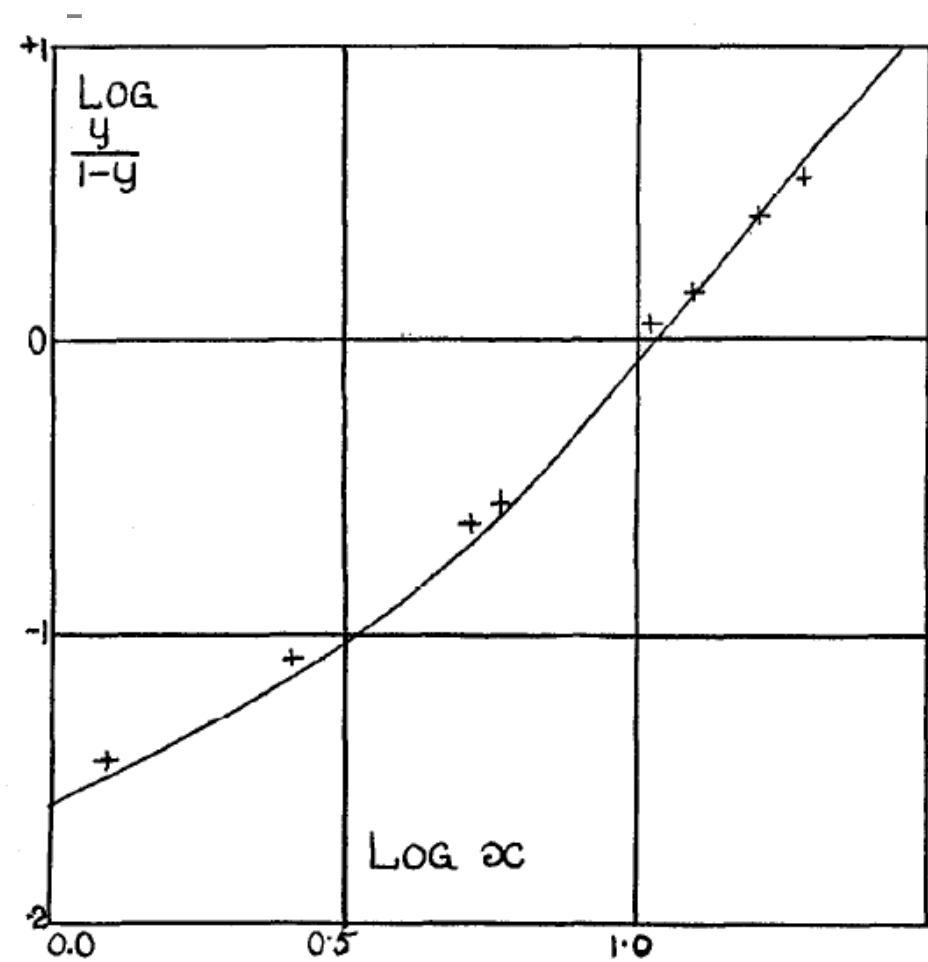


FIG. 2. Test of formula (6). Curve drawn from 6 experimental points from Table IV.



THE HEMOGLOBIN SYSTEM.

VI. THE OXYGEN DISSOCIATION CURVE OF HEMOGLOBIN.*

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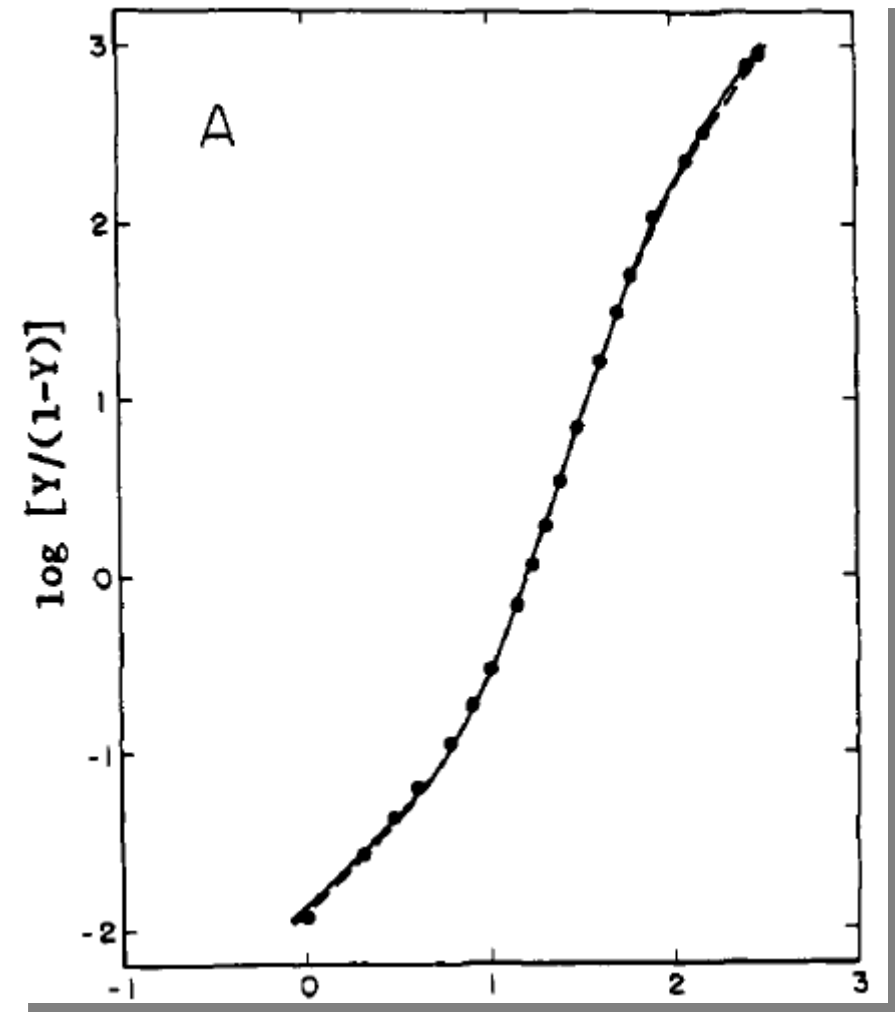
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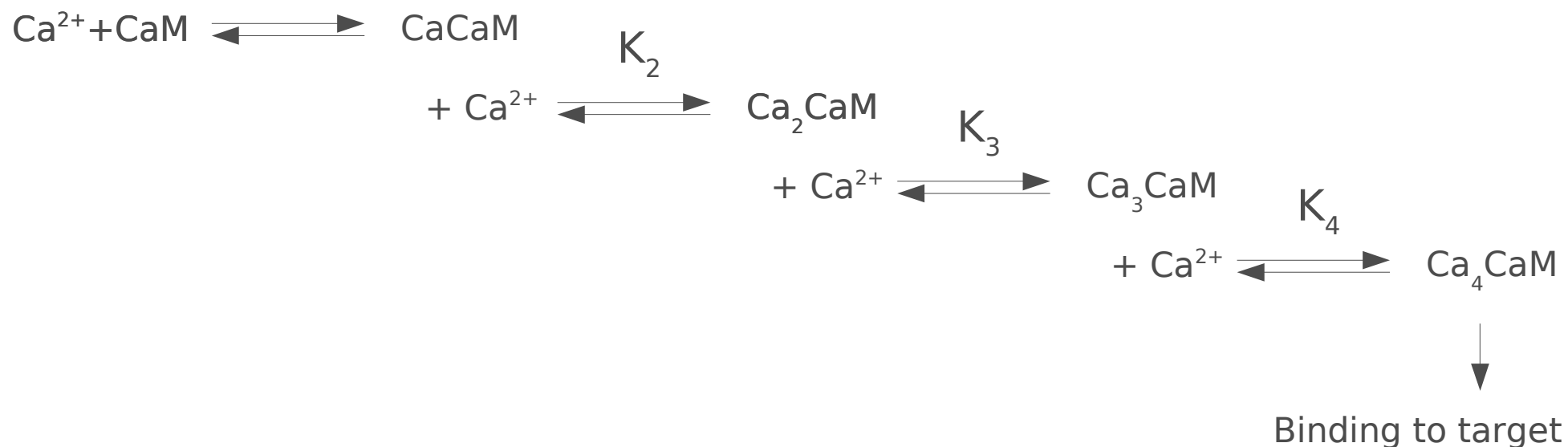
This work gives the oxygen dissociation curves of solutions previously investigated in regard to their acid-binding and base-

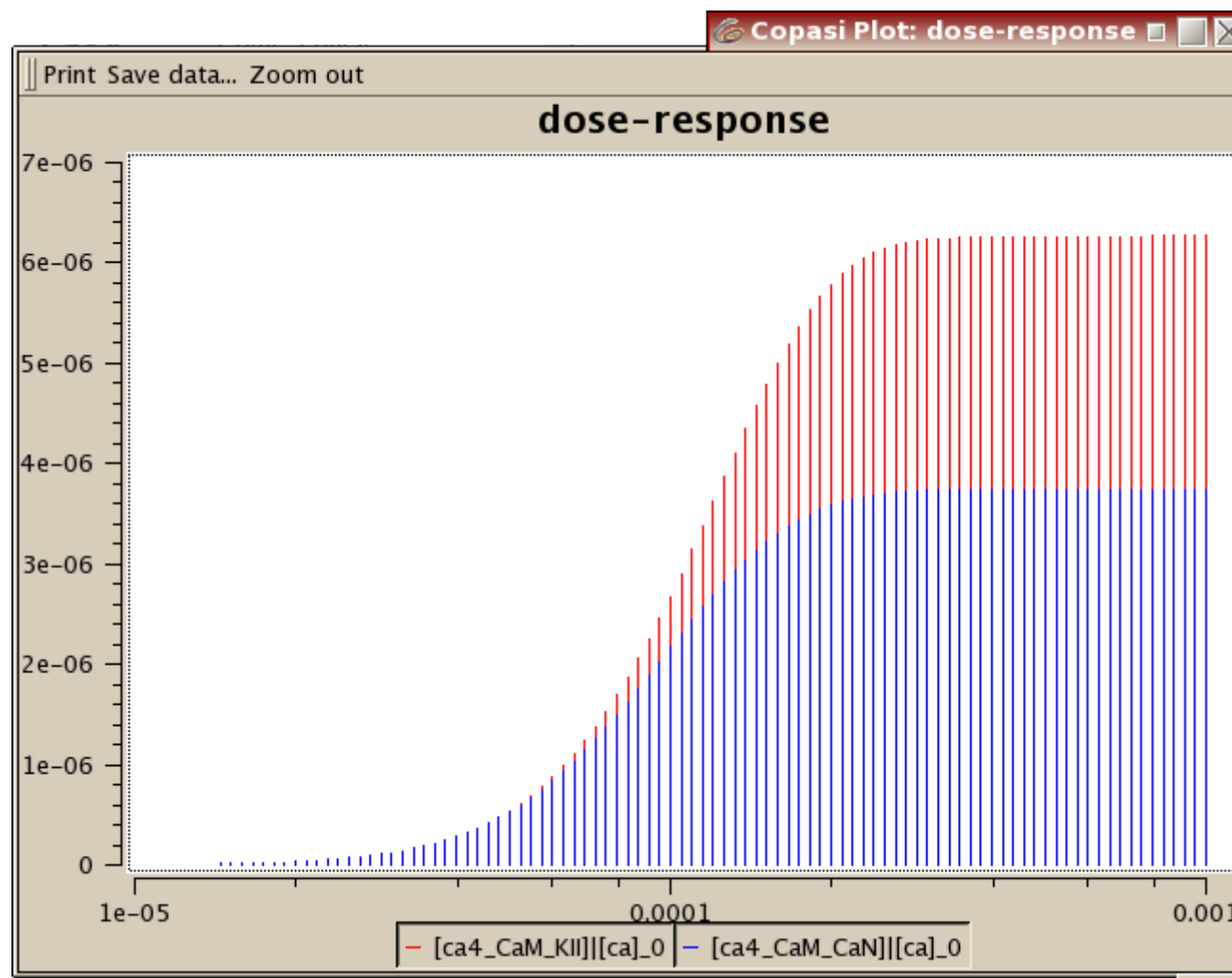
Imai (1973) Analyses of Oxygen Equilibria of Native and Chemically Modified Human Adult Hemoglobins on the Basis of Adair's Stepwise Oxygenation Theory and the Allosteric Model of Monod, Wyman, and Changeux. *Biochemistry* 12: 798-808



Klotz I (1946) The Application of the Law of Mass Action to Binding by Proteins. Interactions with Calcium. *Arch Biochem*, 9:109–117.

$$\bar{Y} = \frac{1}{n} \frac{K_1[\text{Ca}^{2+}] + 2K_1K_2[\text{Ca}^{2+}]^2 + 3K_1K_2K_3[\text{Ca}^{2+}]^3 + 4K_1K_2K_3K_4[\text{Ca}^{2+}]^4}{1 + K_1[\text{Ca}^{2+}] + 2K_1K_2[\text{Ca}^{2+}]^2 + 3K_1K_2K_3[\text{Ca}^{2+}]^3 + 4K_1K_2K_3K_4[\text{Ca}^{2+}]^4}$$



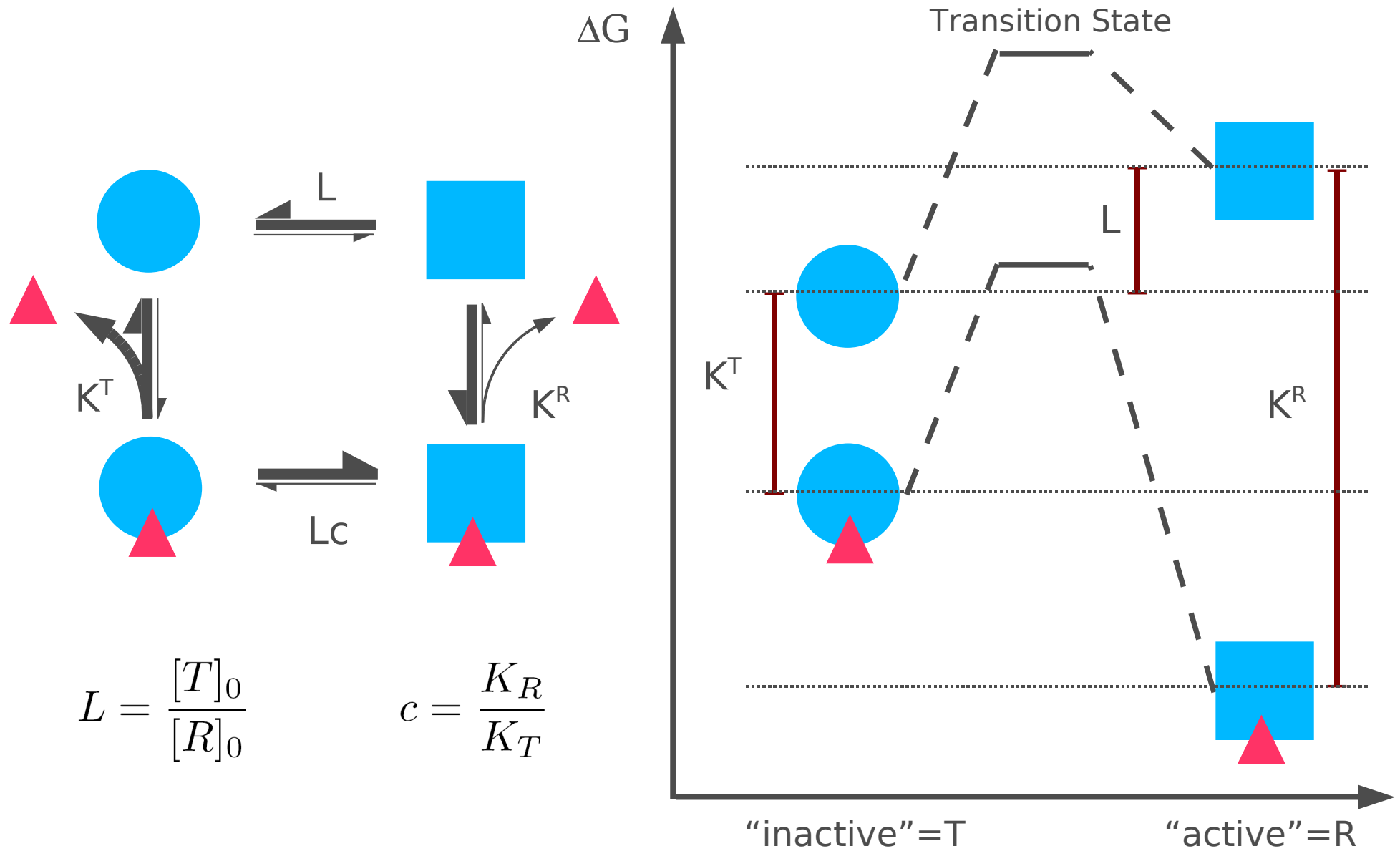


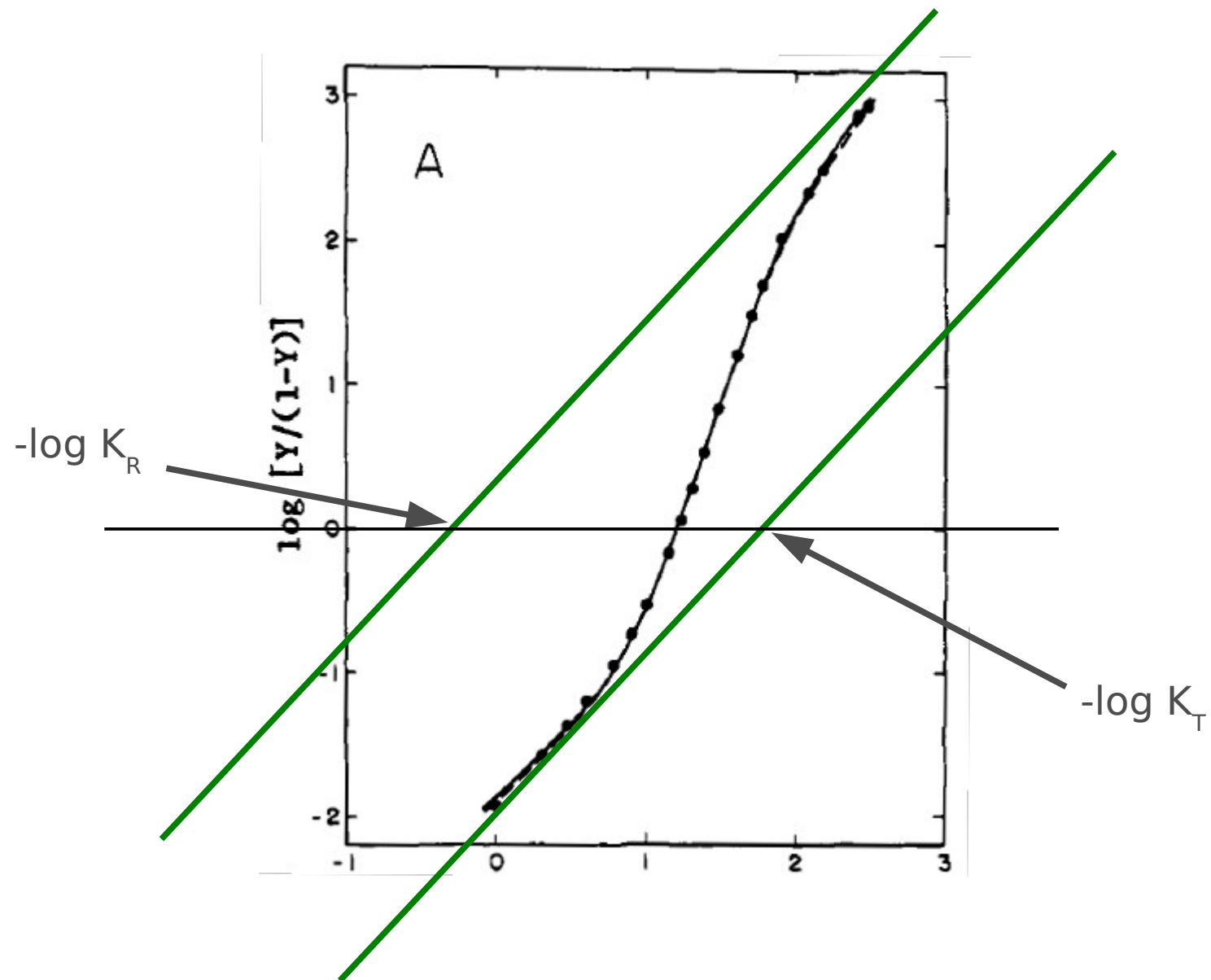
- Calmodulin bound to three calcium ions activates calcineurin
 - Kincaid and Vaughan (1986). *PNAS*, 83: 1193-1197
- Calmodulin bound to two calcium ions can bind CaMKII
 - Shifman et al (2006). *PNAS*, 103: 13968-13973
- Calmodulin affinity for calcium increases once bound to CaMKII
 - Shifman et al (2006) [but many previous reports on other targets: e.g. Burger et al (1983). *JBC*, 258: 14733-14739 ; Olwin et (1984). *JBC* 259: 10949-10955]
- Calcium activates both LTP and LTD through calmodulin
 - Lisman (1989) *PNAS*, 86: 9574-9578
 - High $[Ca^{2+}]$ (high freq) $\cong$ CaMKII ; Low $[Ca^{2+}]$ (low freq) $\cong$ Calcineurin
- What is the cause of the increasing affinity? How do binding sites communicate?
- What is the relationship between the known structures and the quantitative model? What causes the structural transitions?



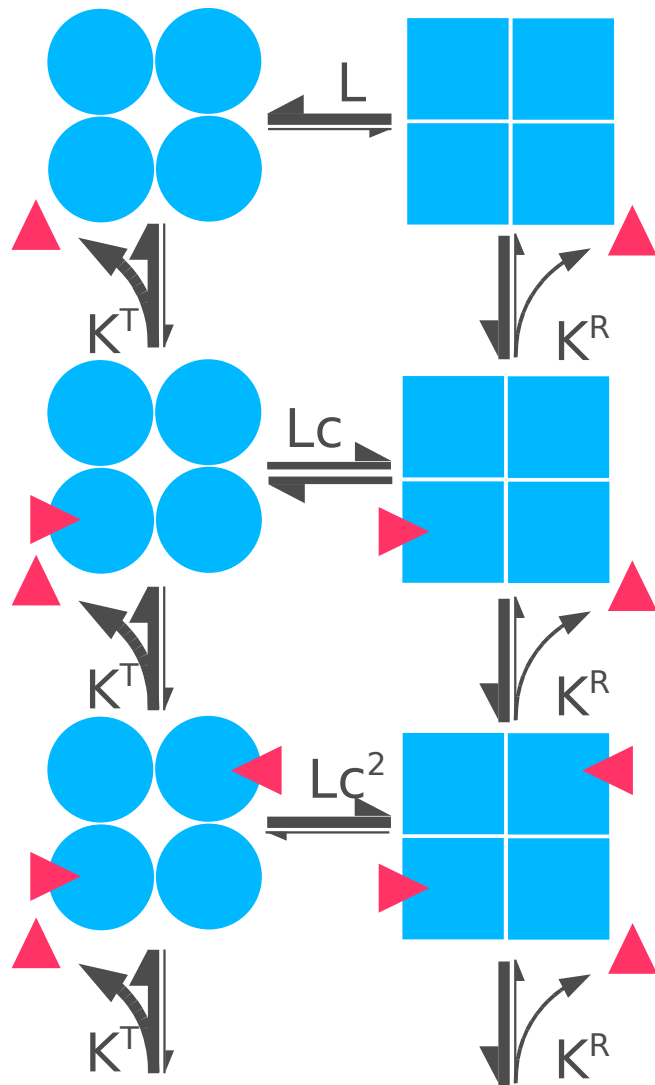
- Monod, Wyman, Changeux (1965). On the nature of allosteric transitions: a plausible model. *J Mol Biol*, 12: 88-118

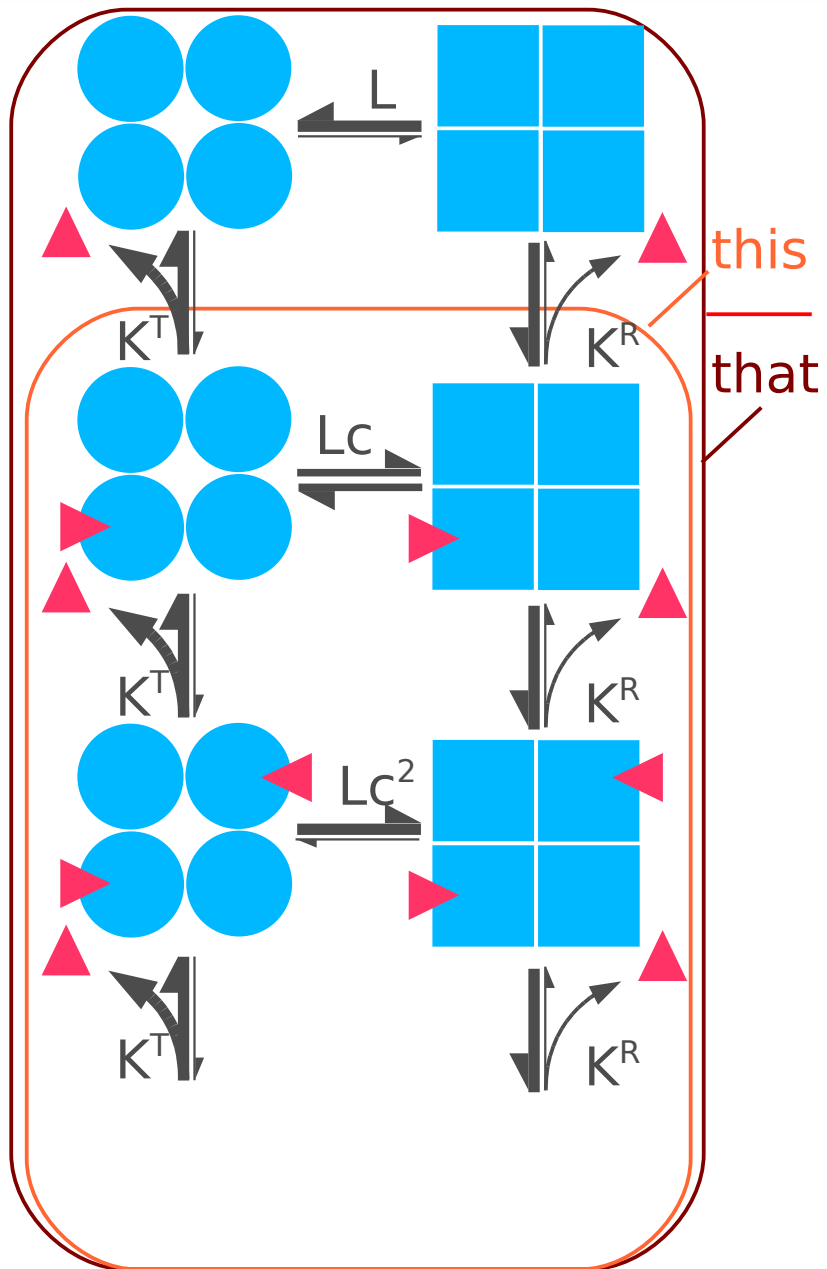


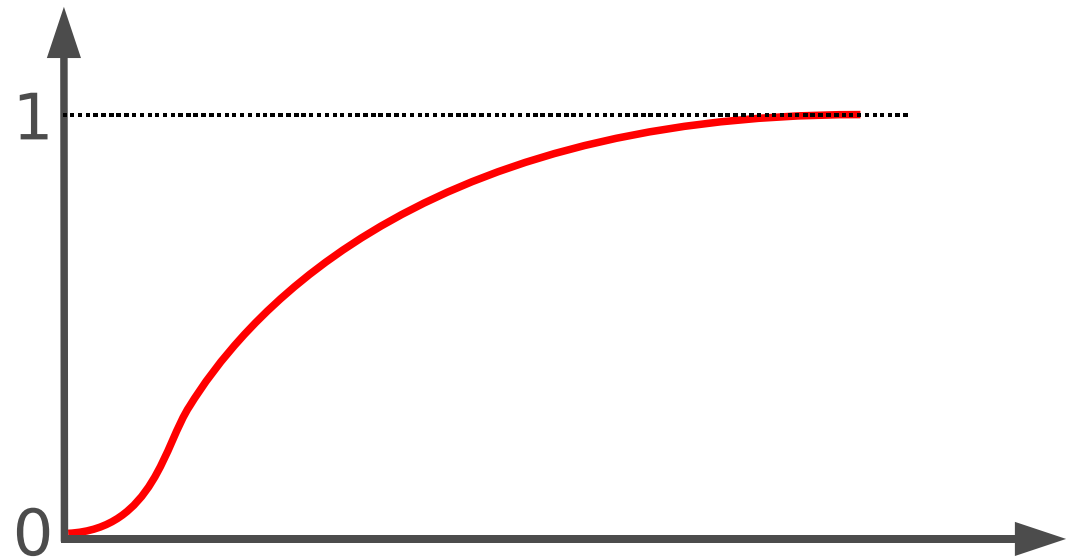
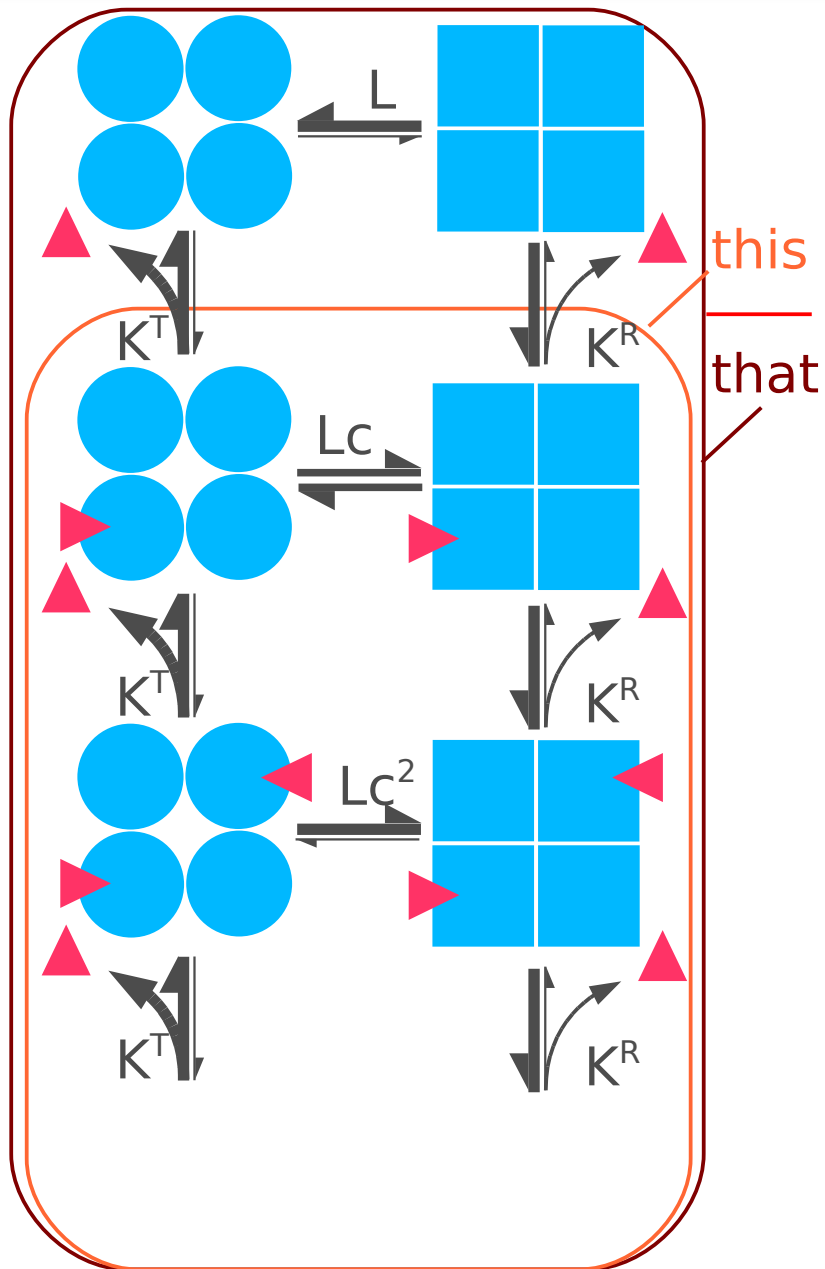


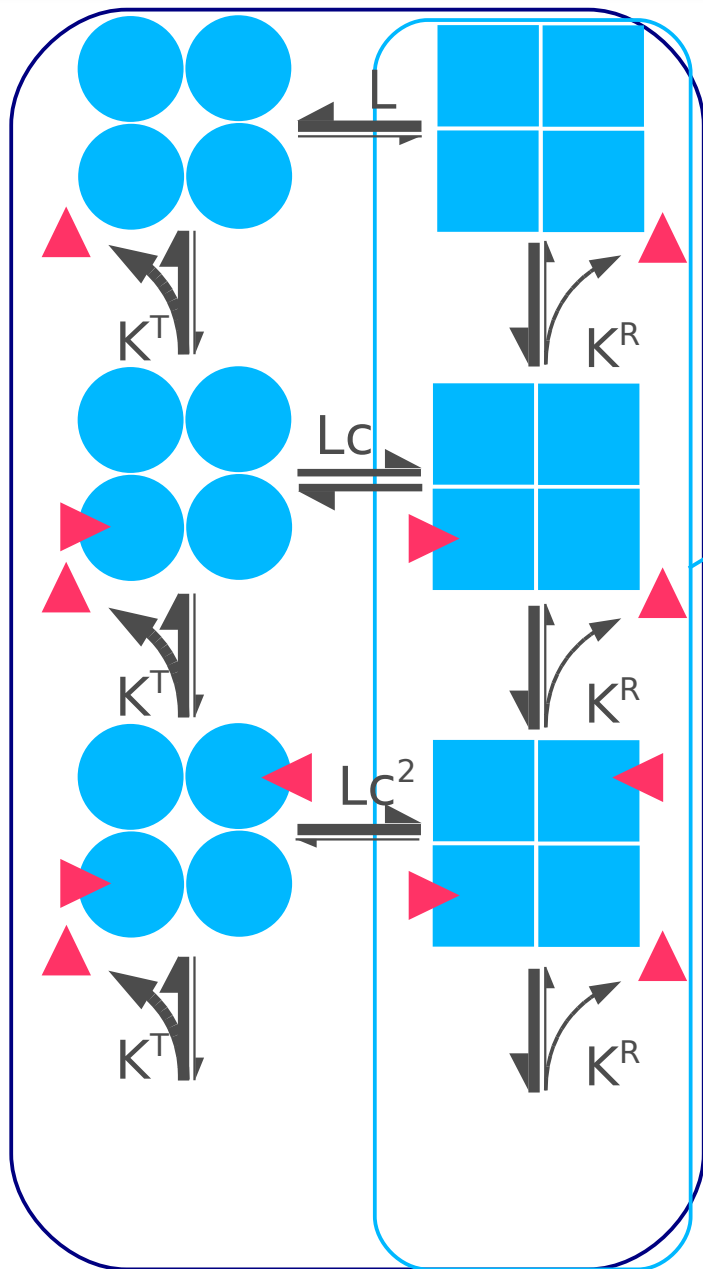




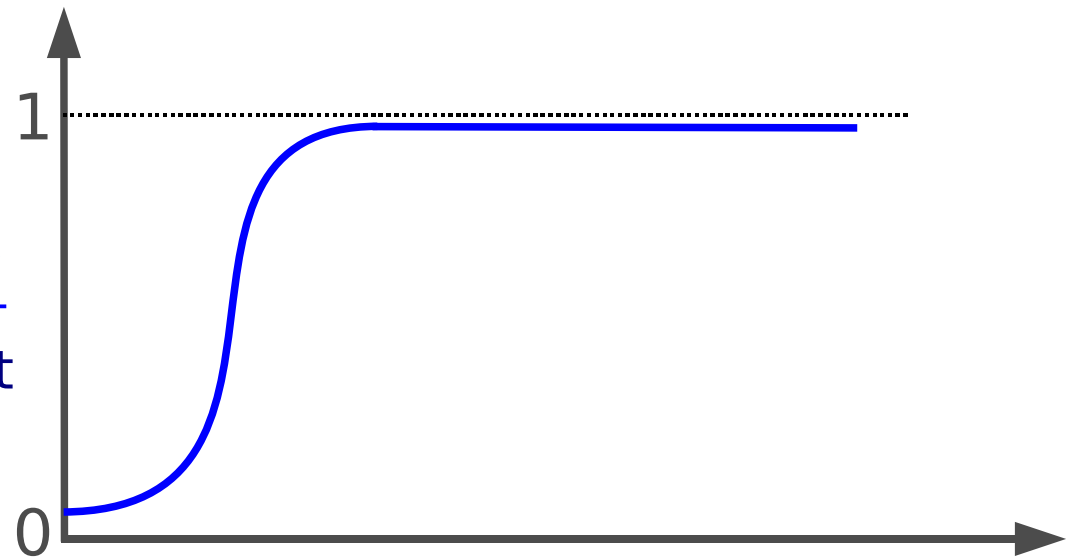


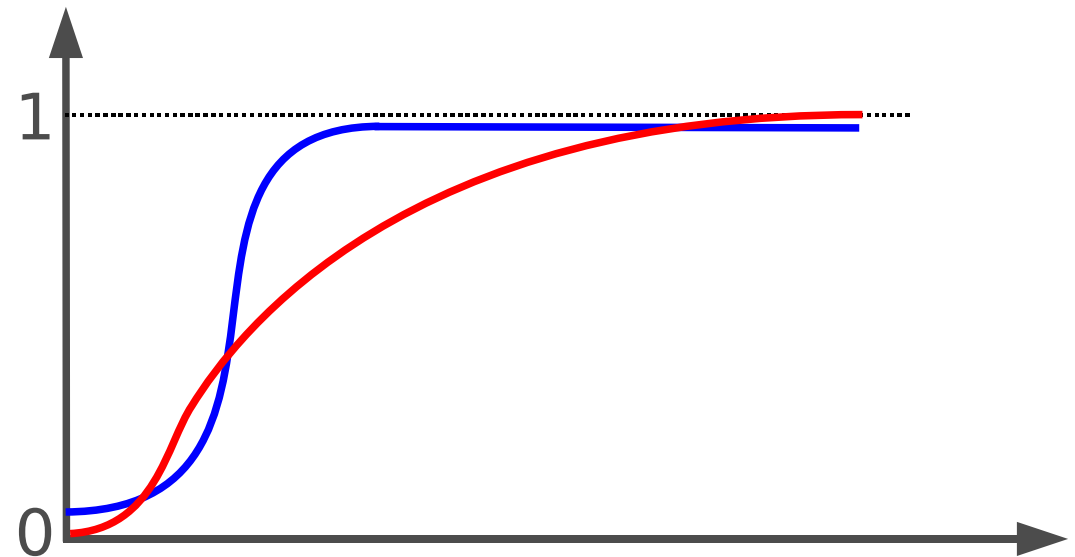
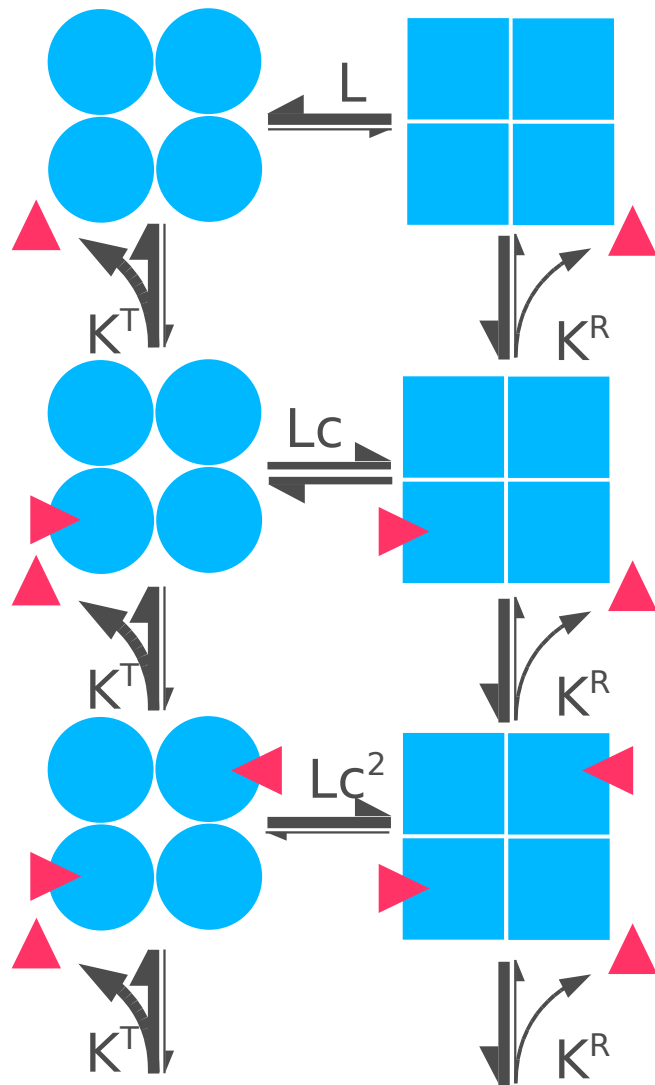


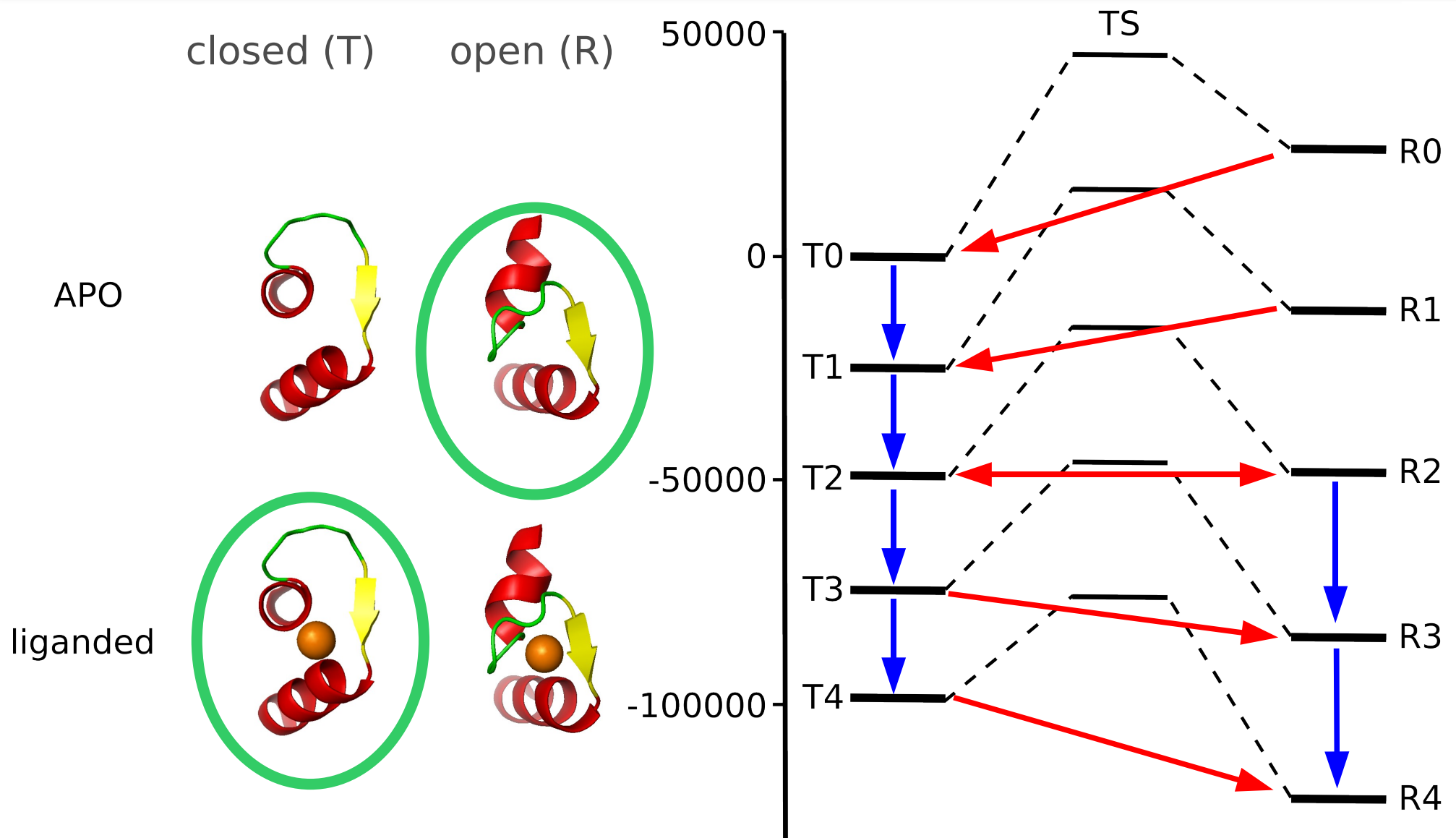




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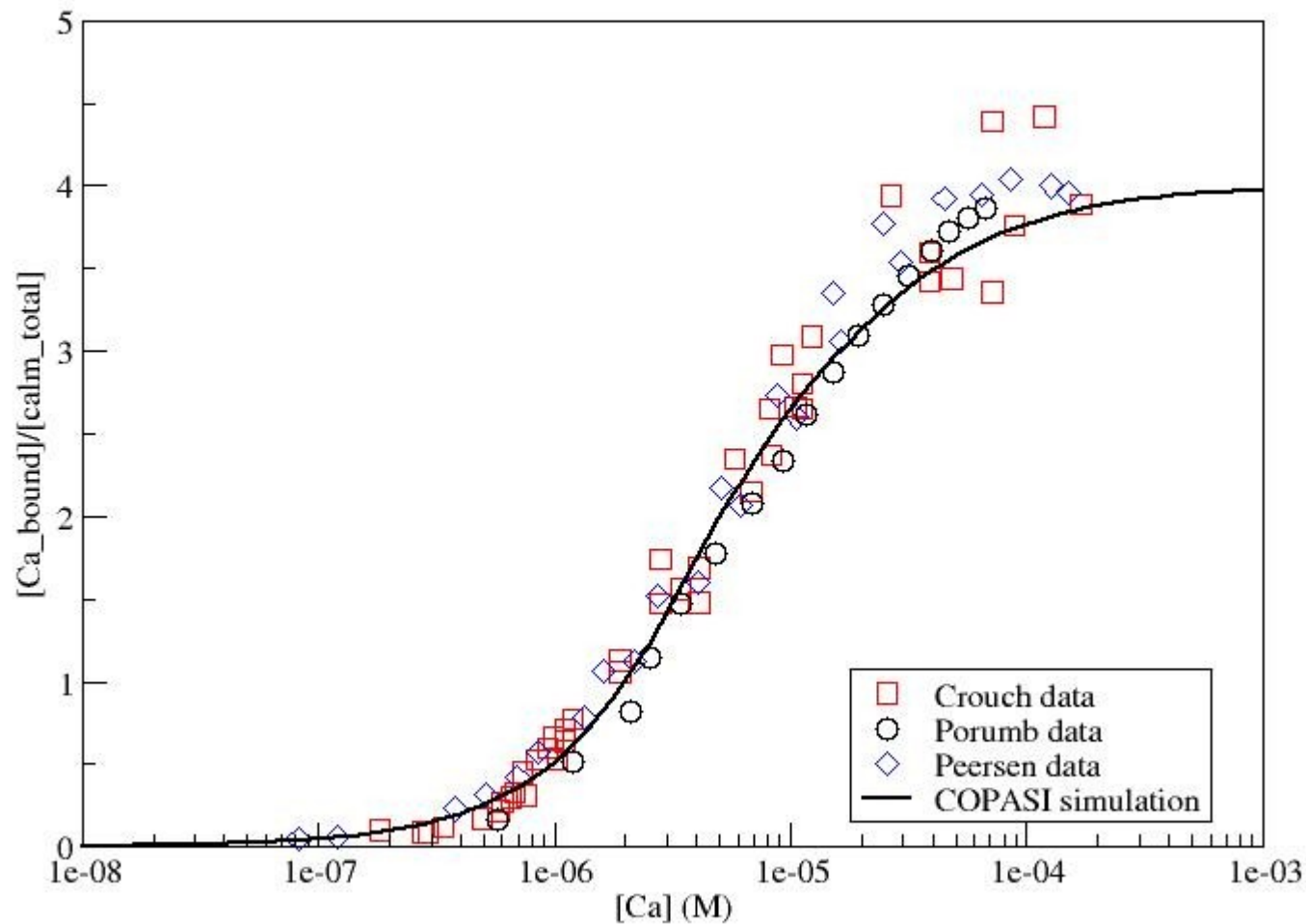


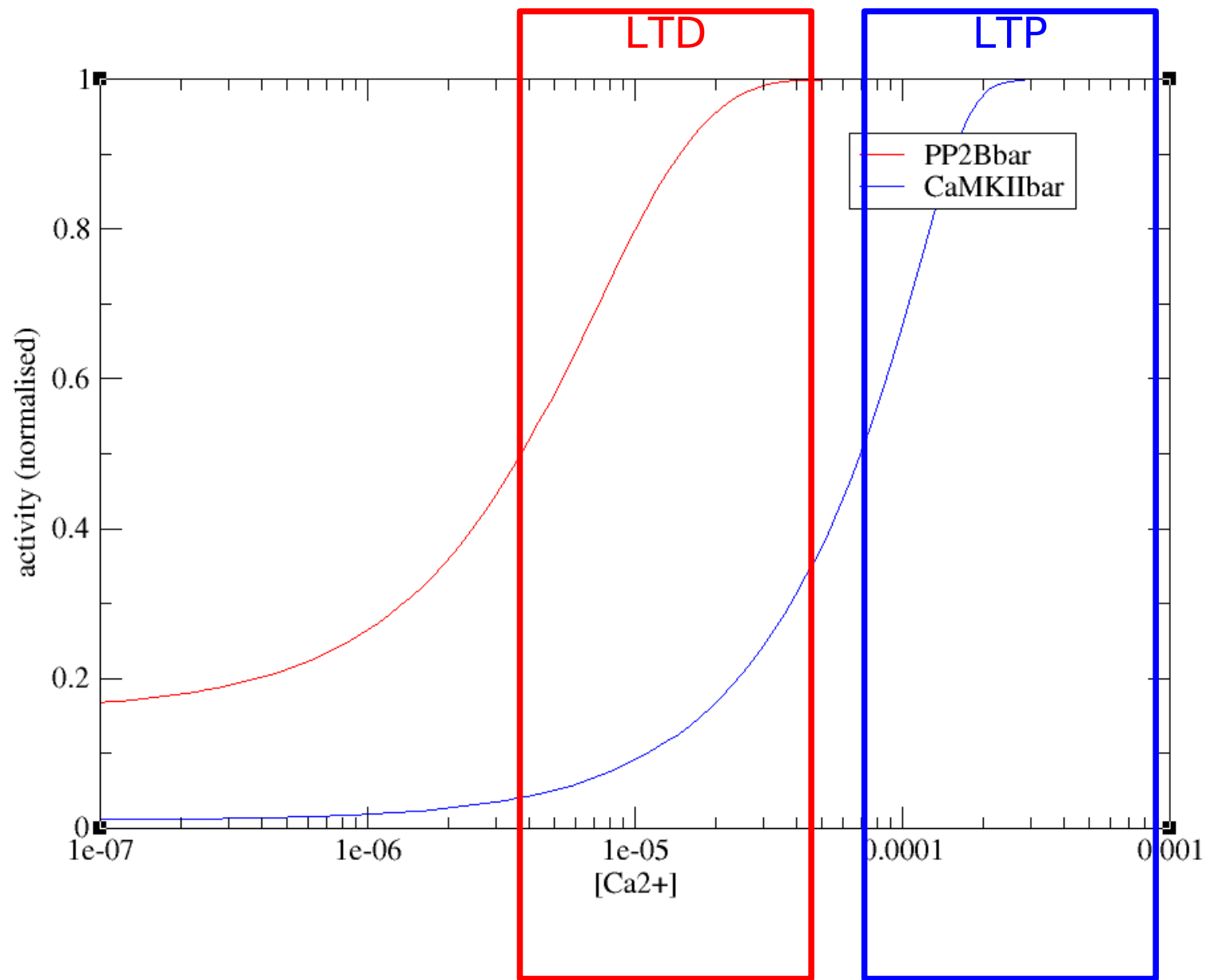


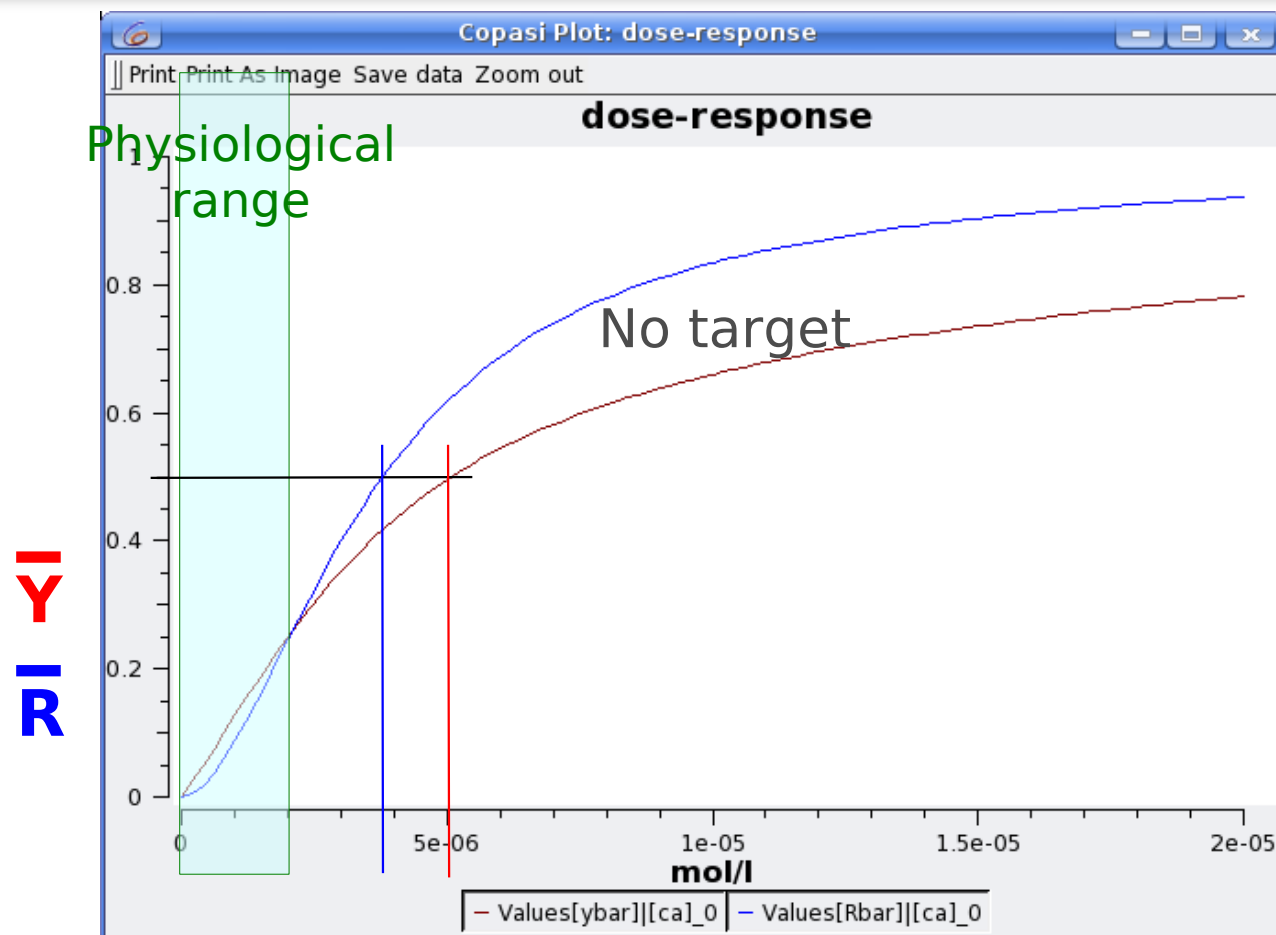


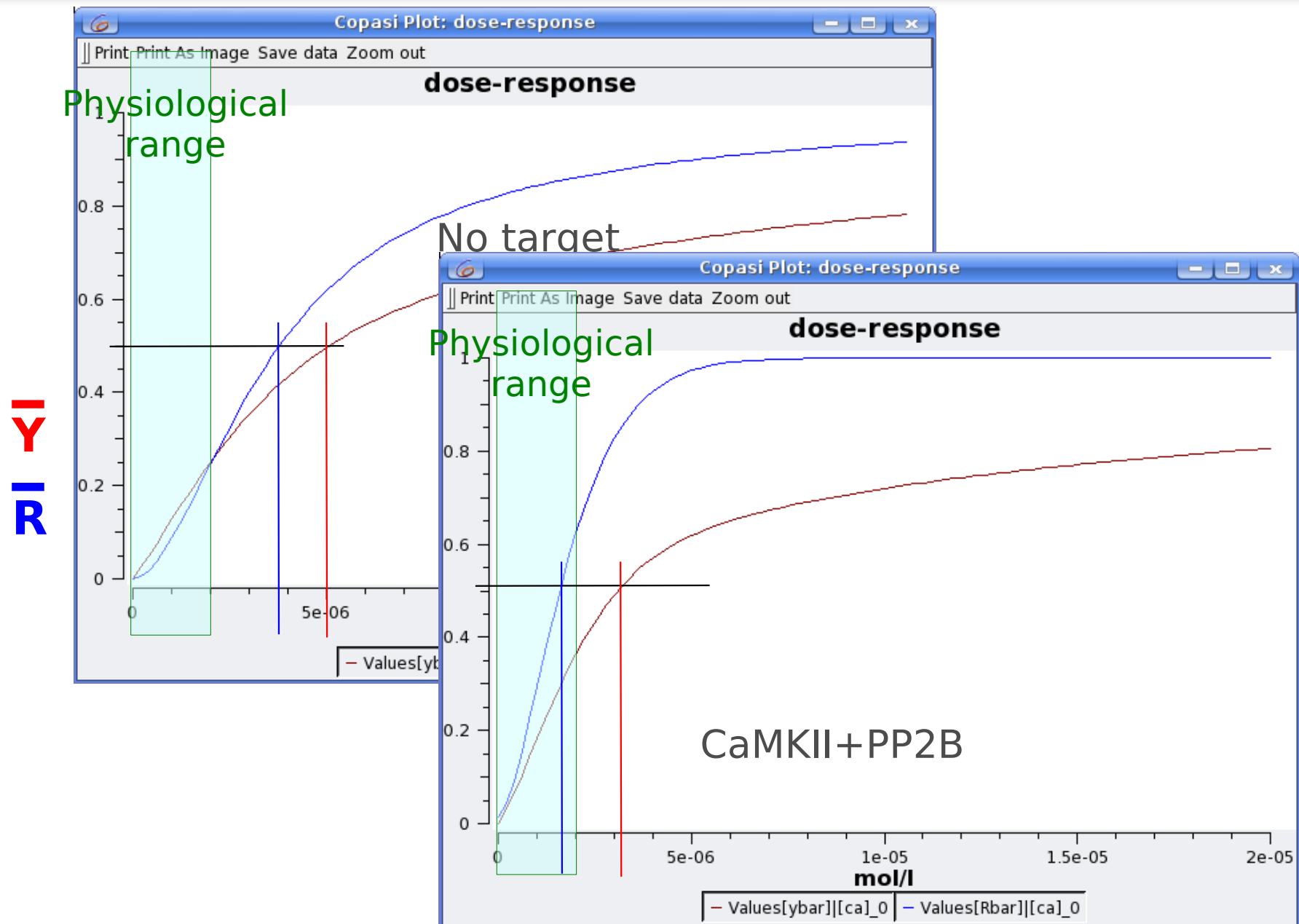
Stefan MI, Edelstein SJ, Le Novère N (2008) An allosteric model of calmodulin explains differential activation of PP2B and CaMKII. *Proc Natl Acad Sci USA*, 105:10768-10773

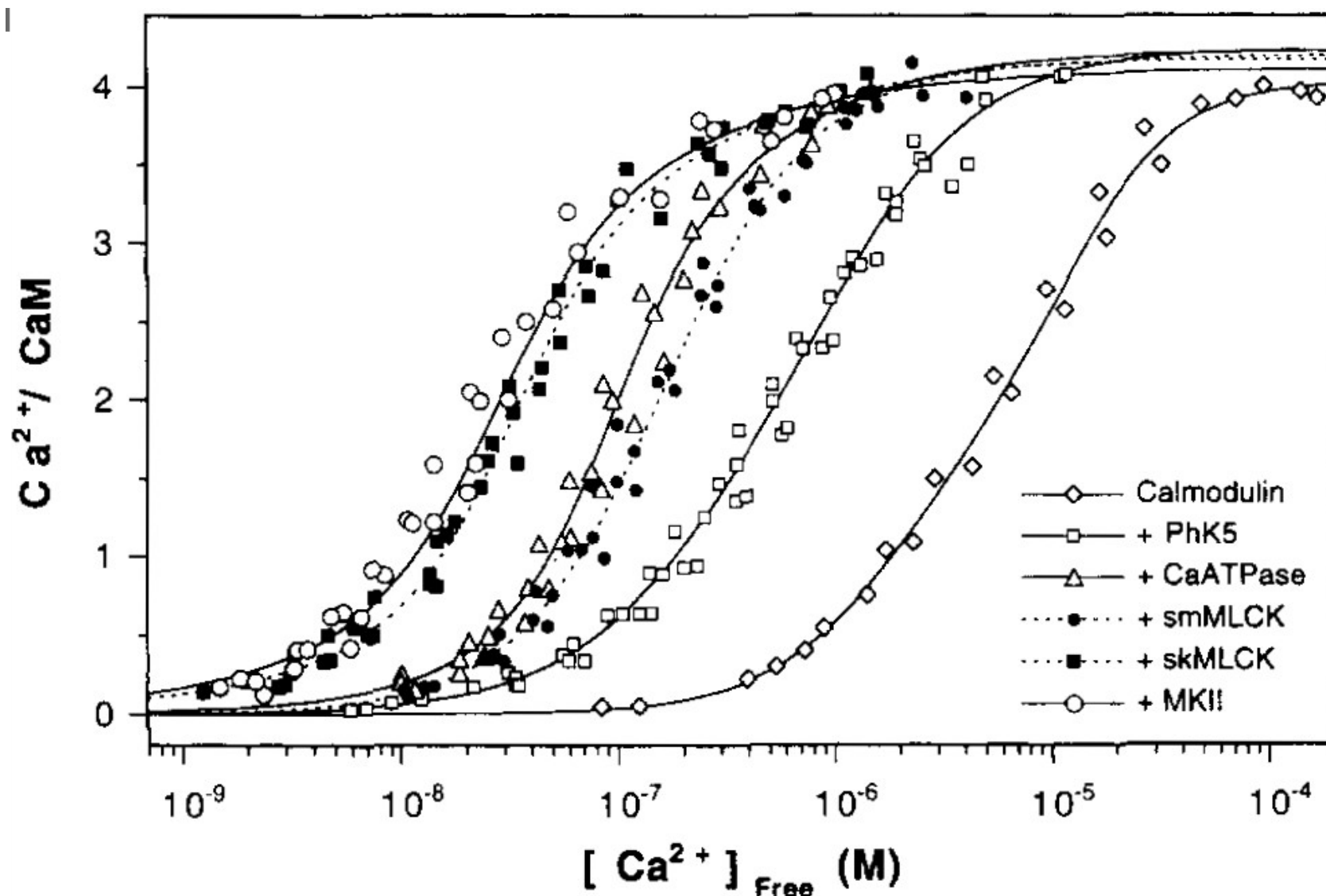






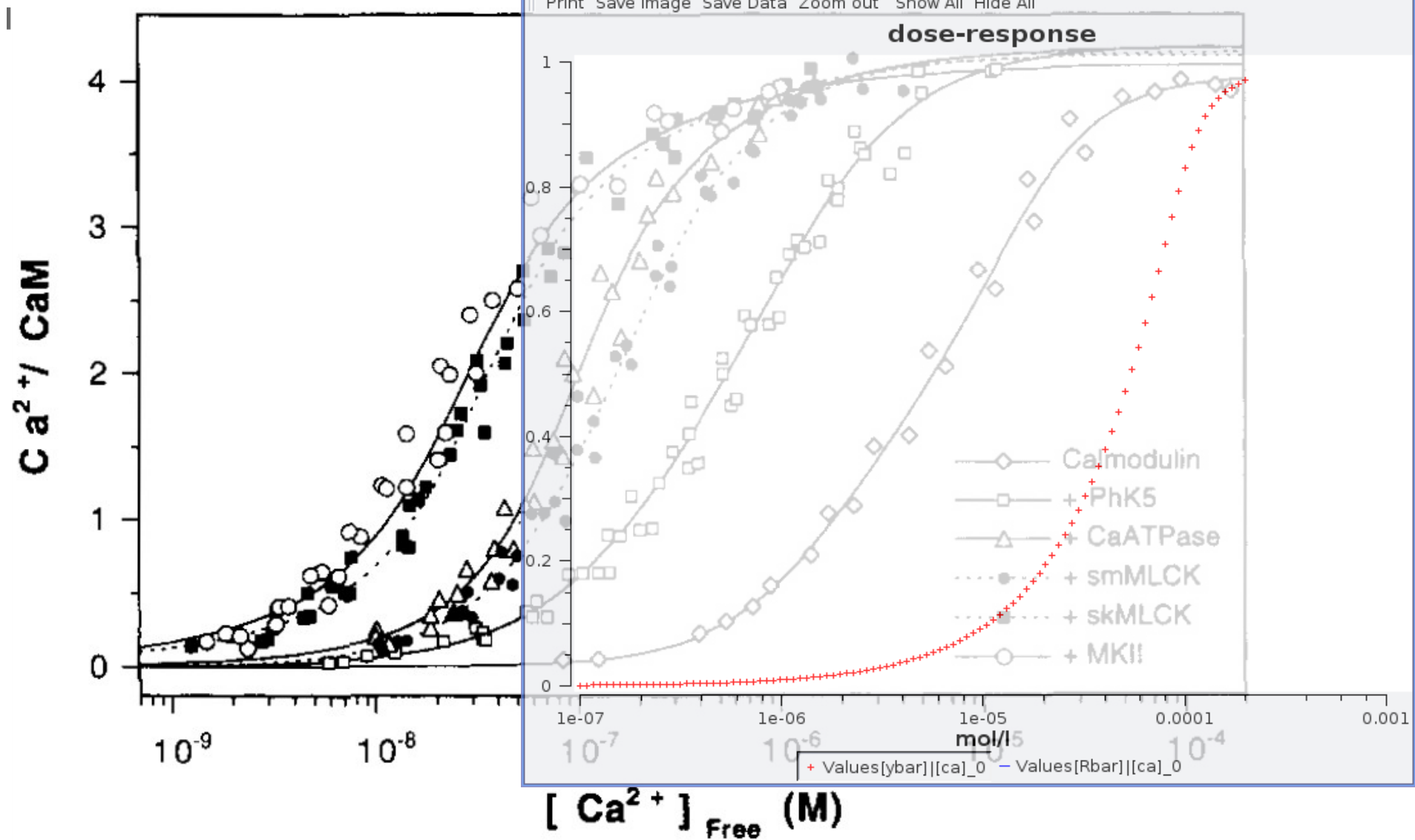


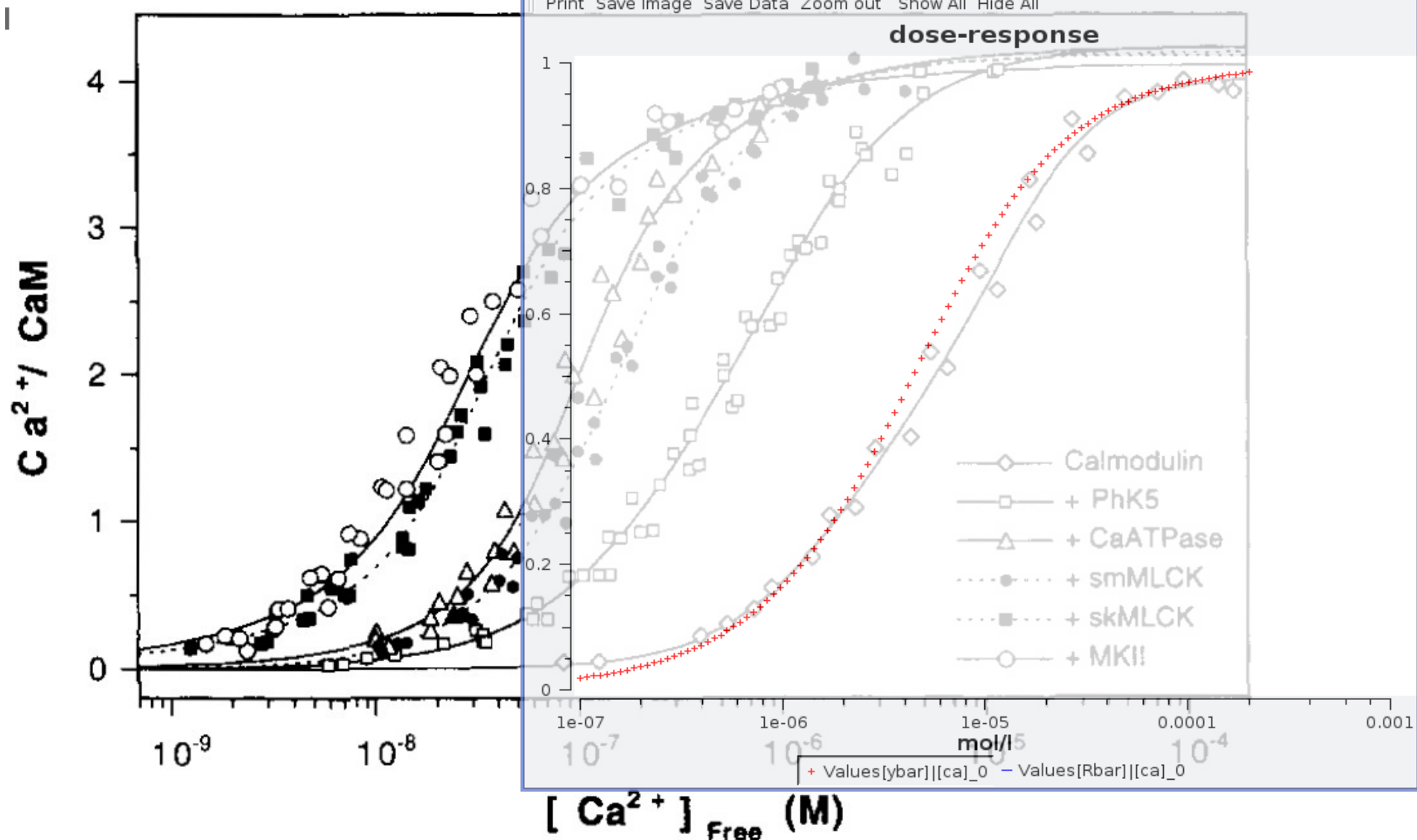




Peersen et al. (1997) Intermolecular tuning of calmodulin by target peptides and proteins: Differential effects on Ca^{2+} binding and implications for kinase activation *Prot Sci*, 6: 794-807

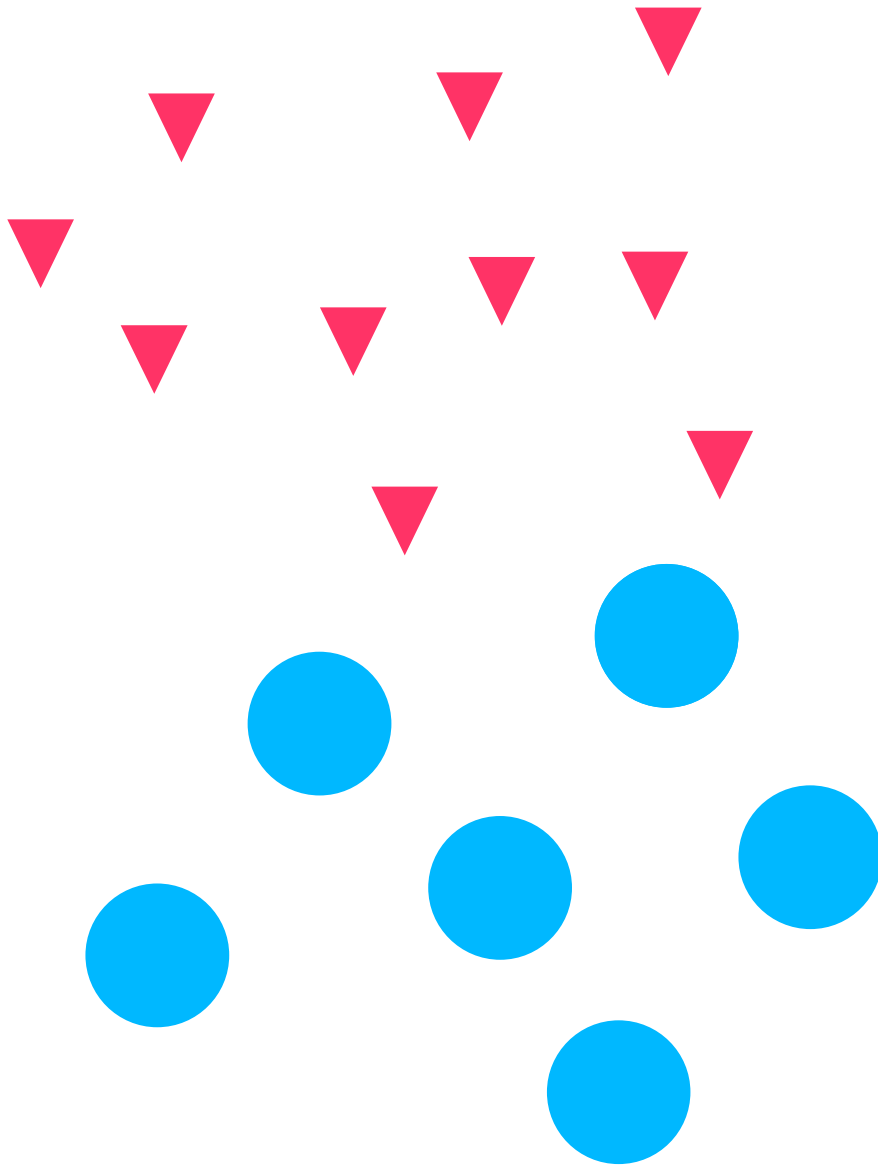




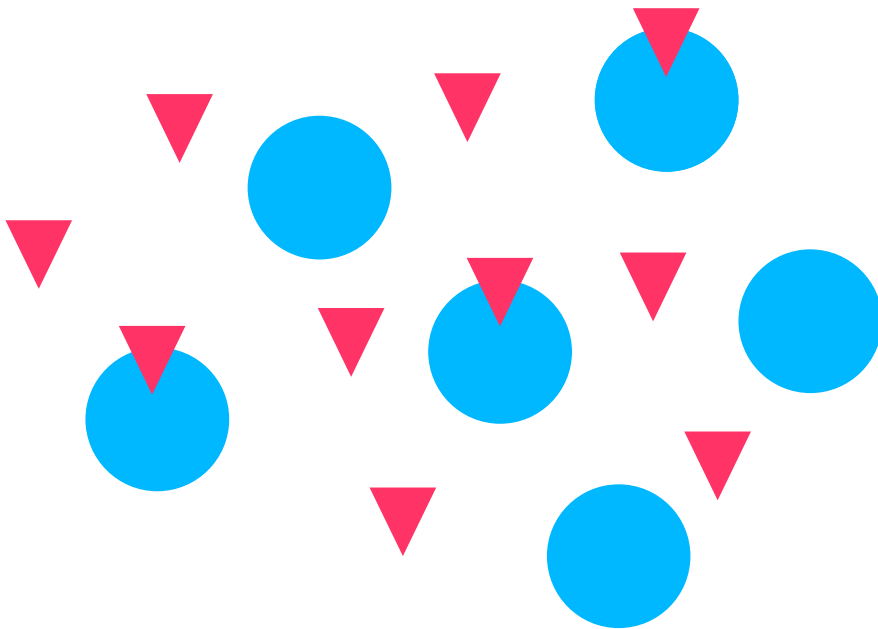
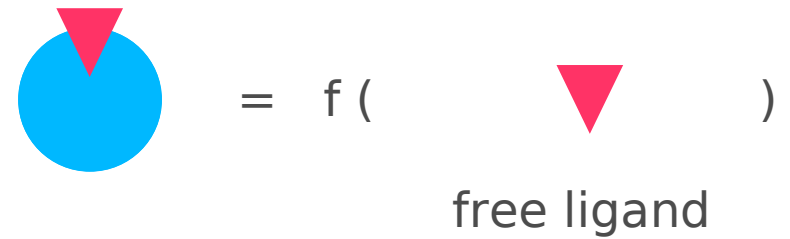


Edelstein SJ, Stefan MI, Le Novère N. Ligand depletion *in vivo* modulates the dynamic range of cooperative signal transduction. *submitted*

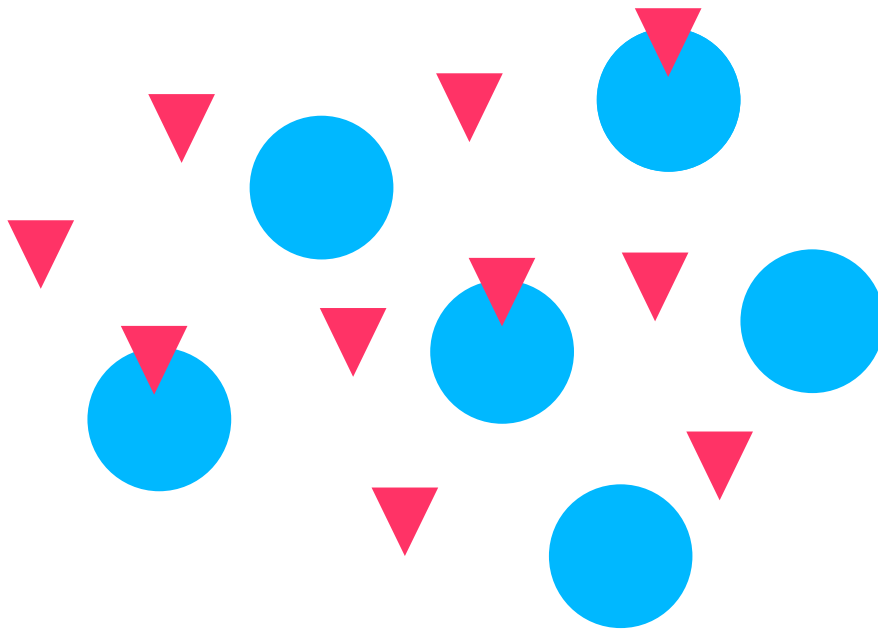
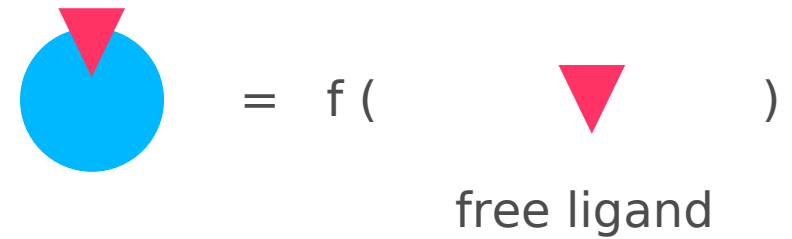




Chemistry (mass-action law)

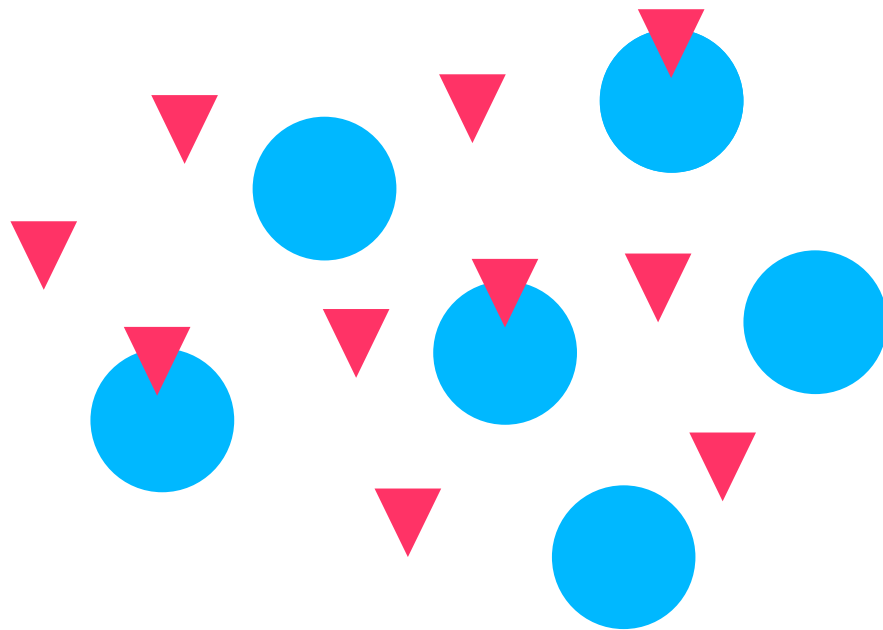


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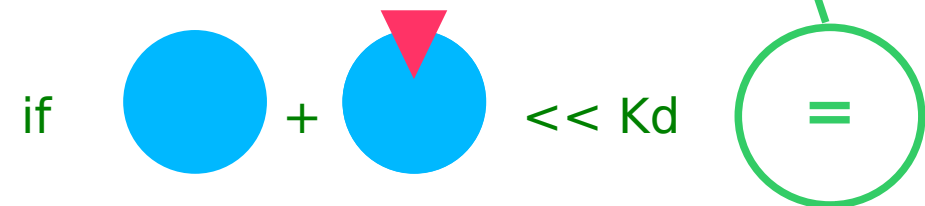
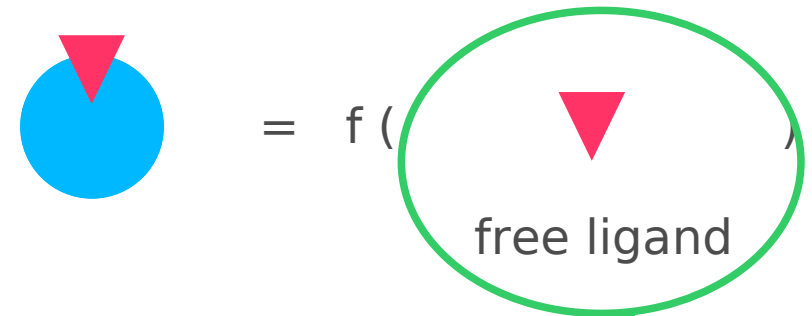


Cellular signalling

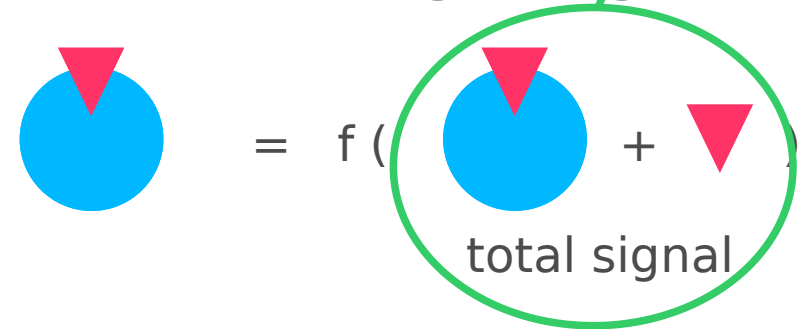




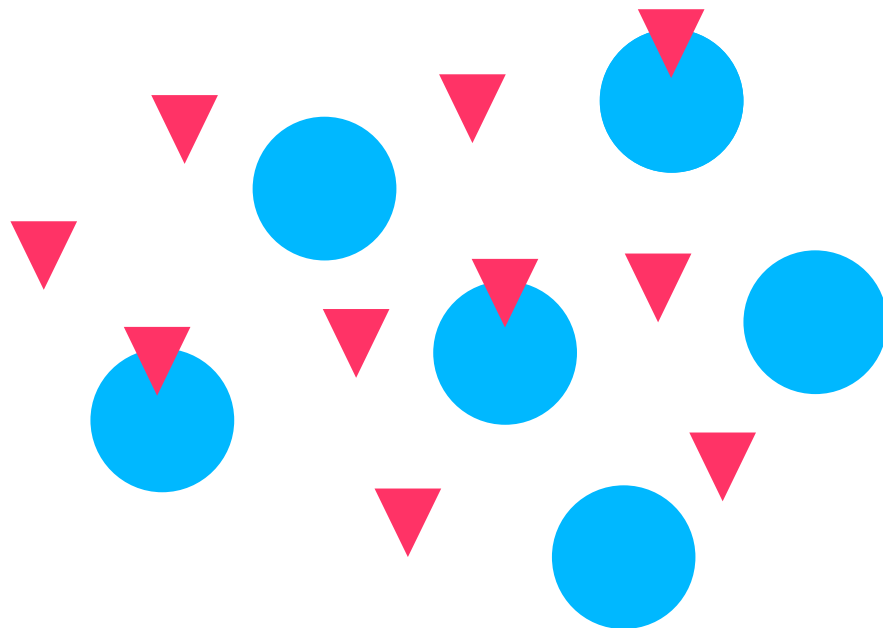
Chemistry (mass-action law)



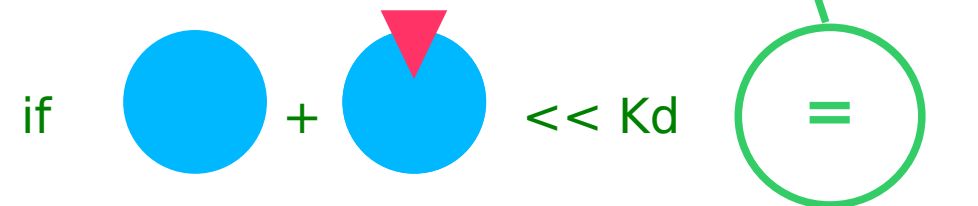
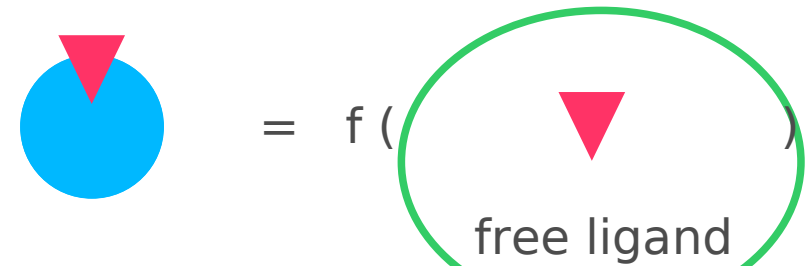
Cellular signalling



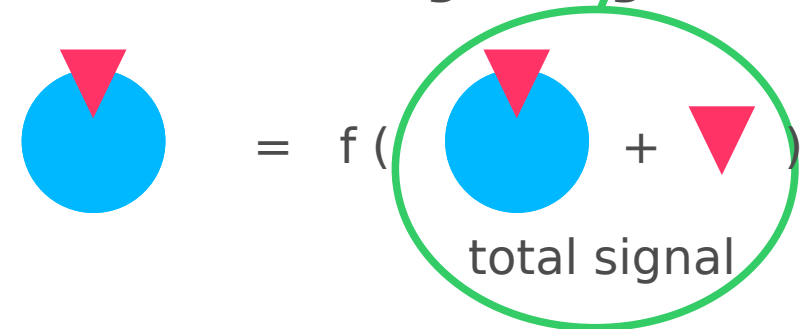
This is generally not the case in signalling:
Concentrations of sensor are in micromolar
range, as are the dissociation constants



Chemistry (mass-action law)

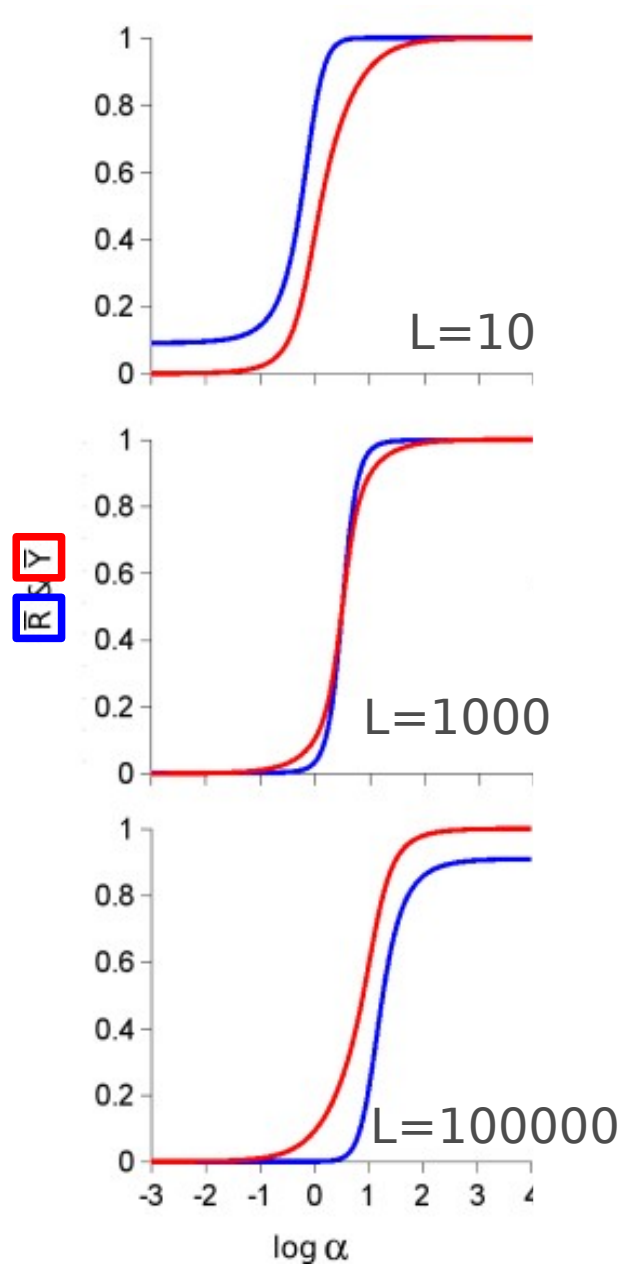


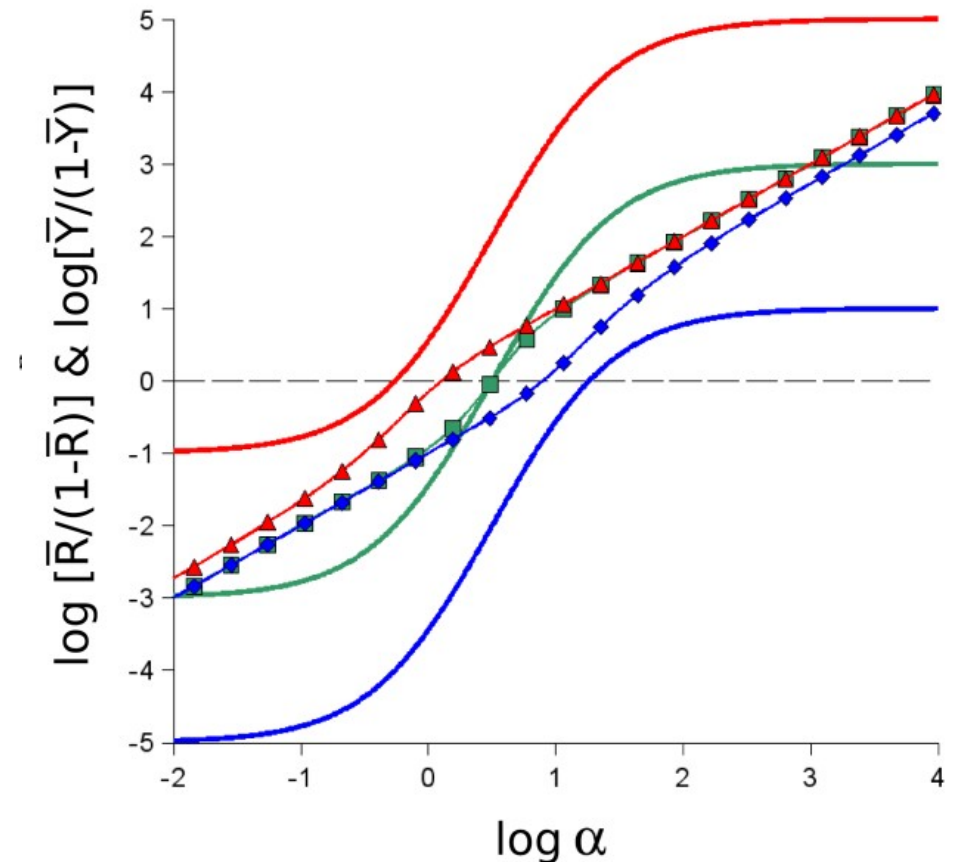
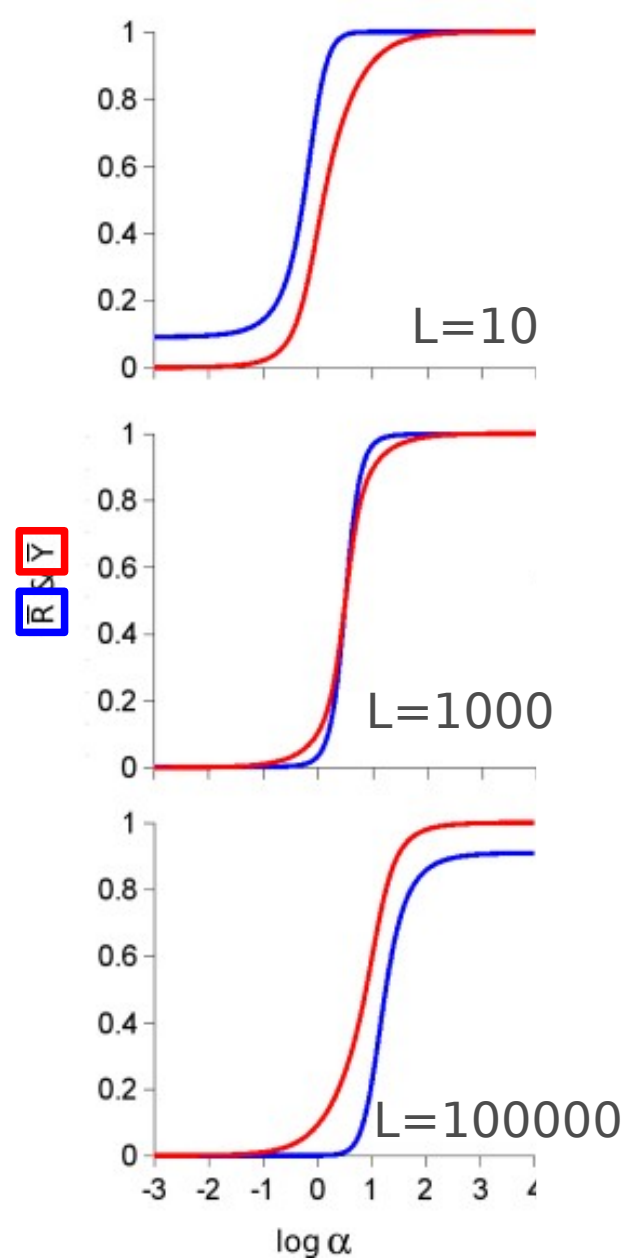
Cellular signalling

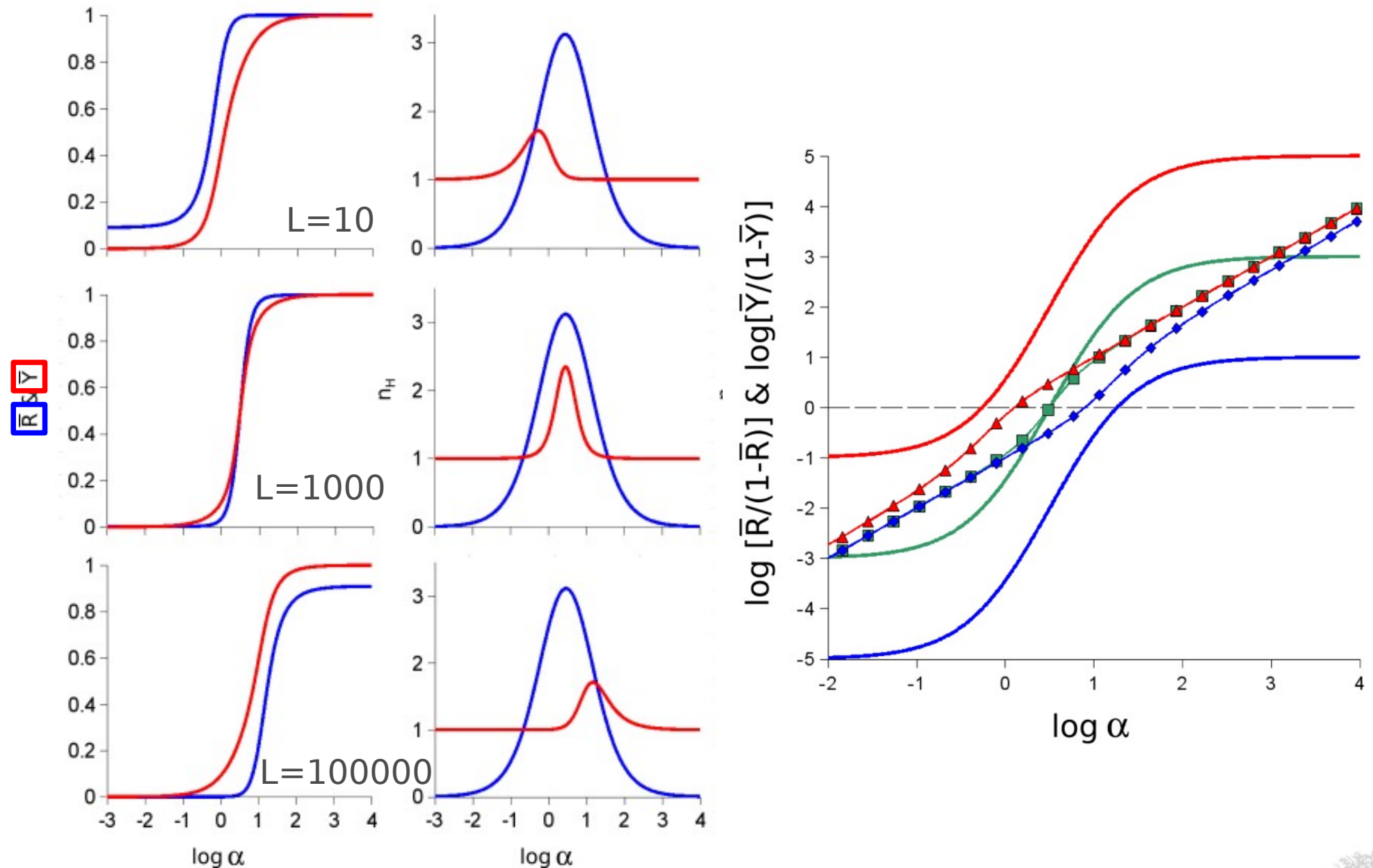


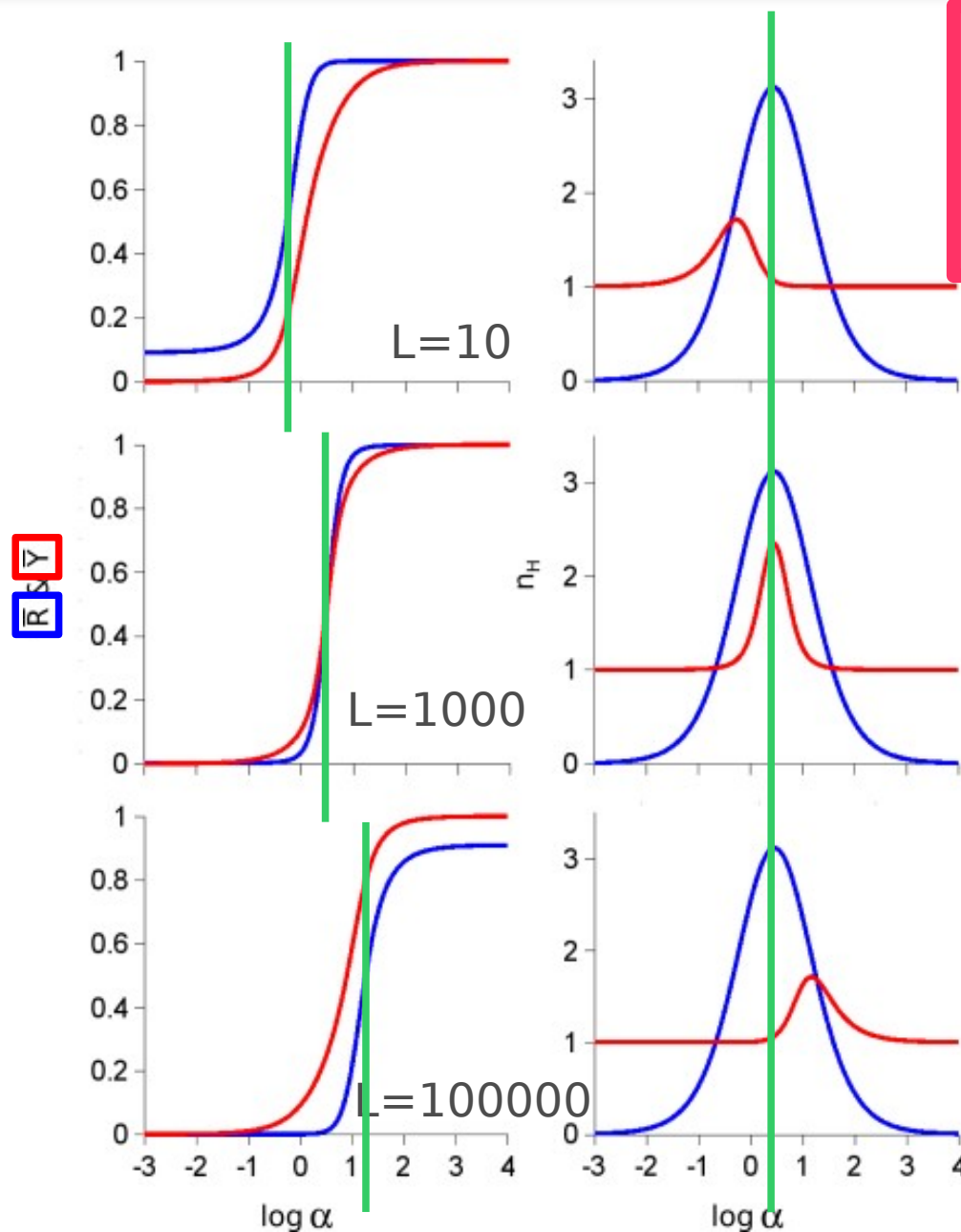


Hill number not suitable for state function

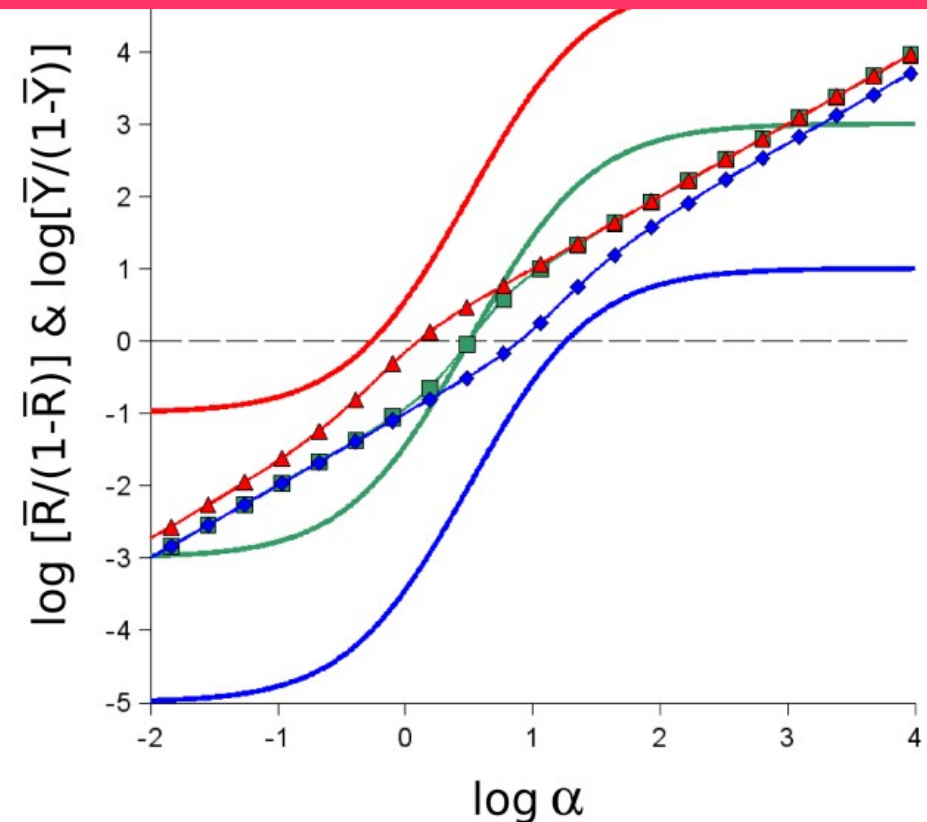


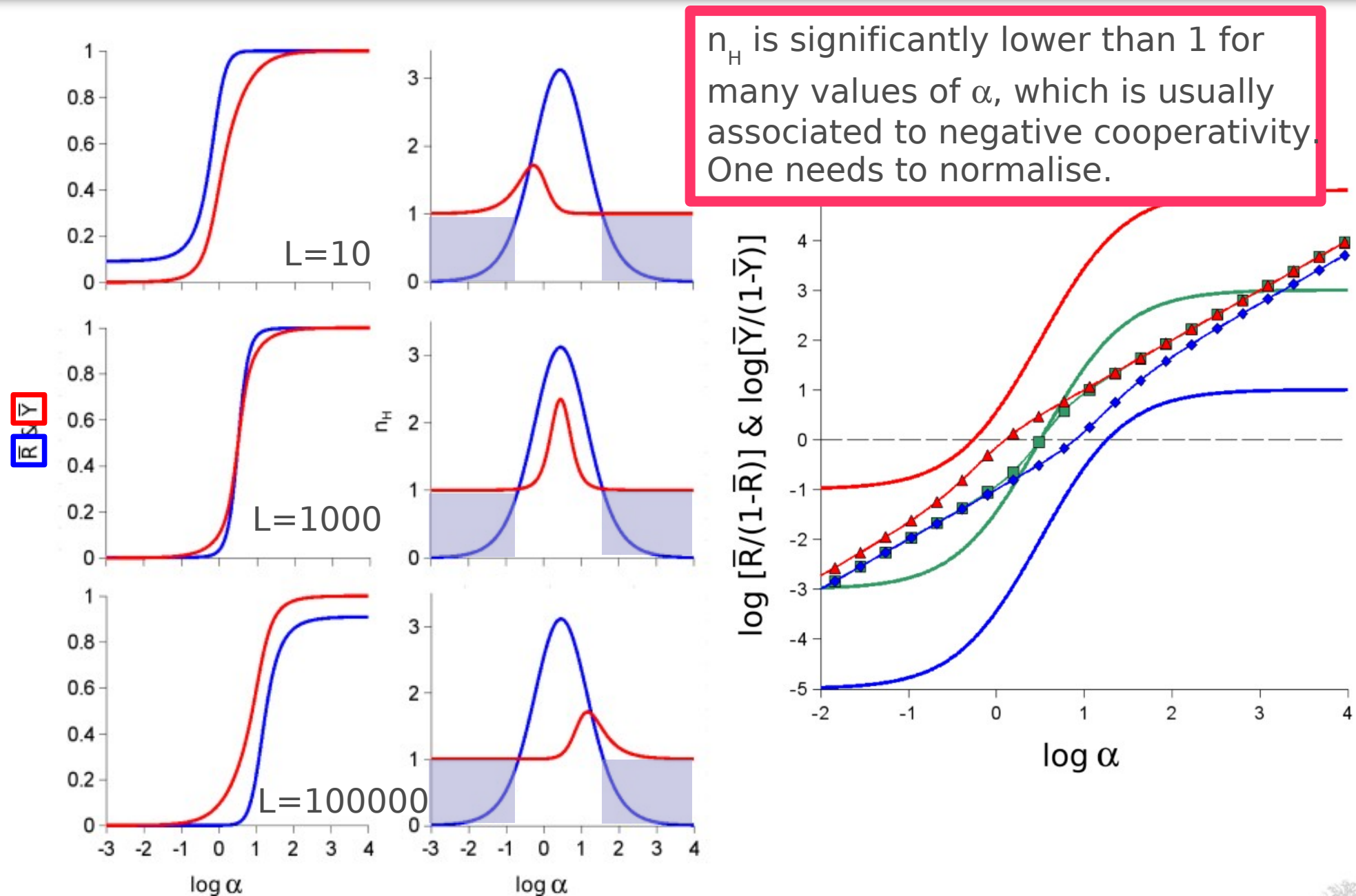


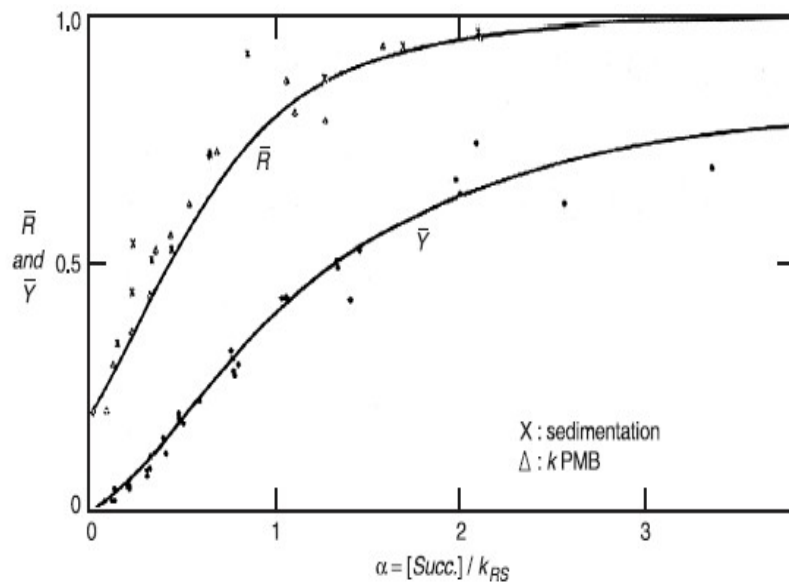




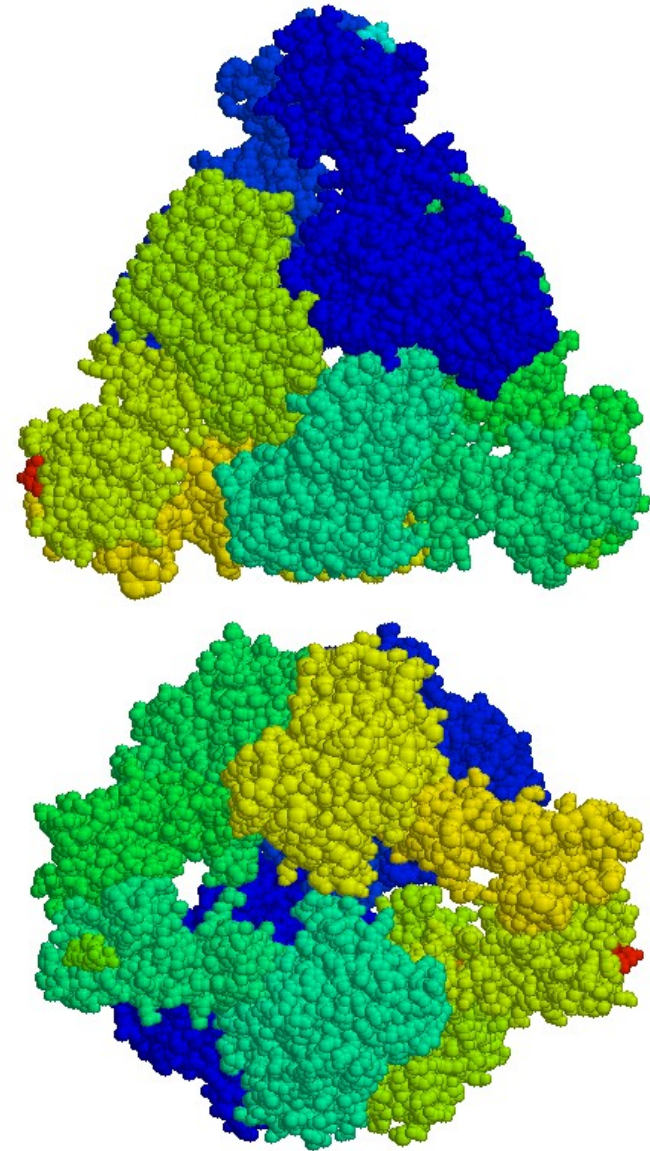
The maximum value of n_H does not depend on the free energy of conformational transition and does correspond to the maximal cooperativity

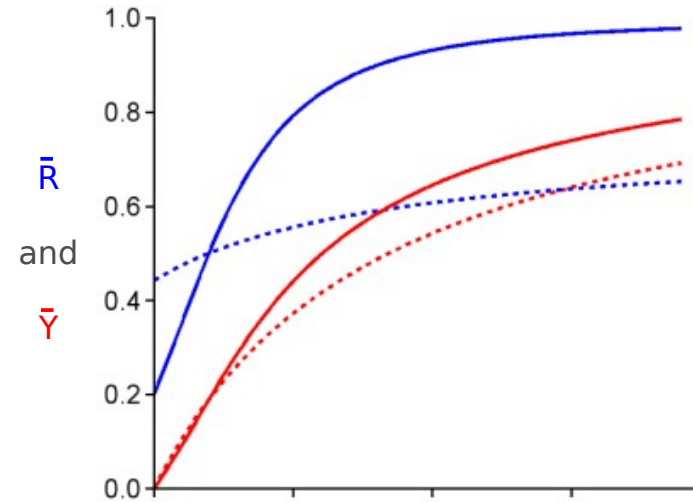
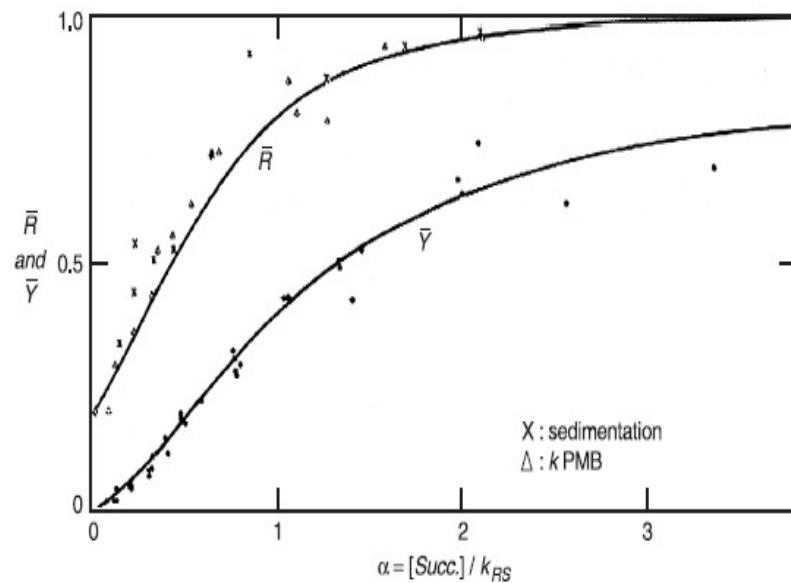






Changeux and Rubin (1968) Allosteric Interactions in Aspartate Transcarbamylase. III. Interpretation of Experimental Data in Terms of the Model of Monod, Wyman, and Changeux. *J Biol Chem* 63: 529

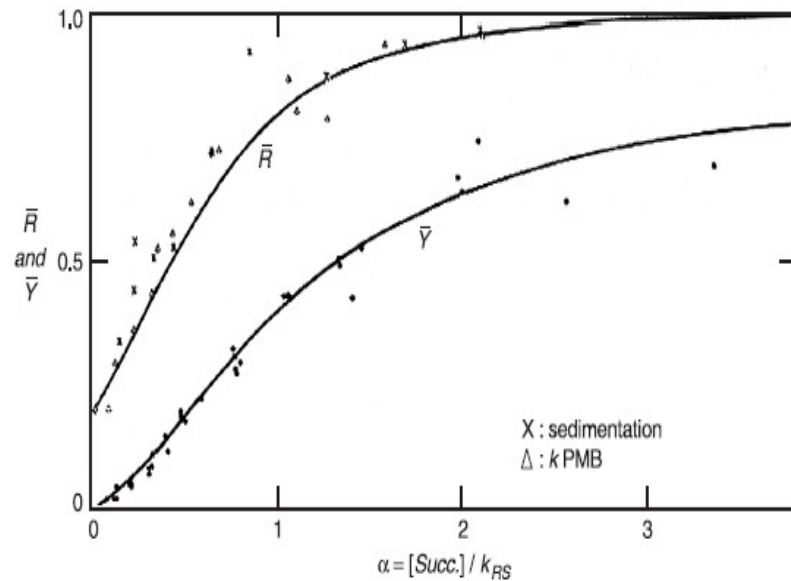




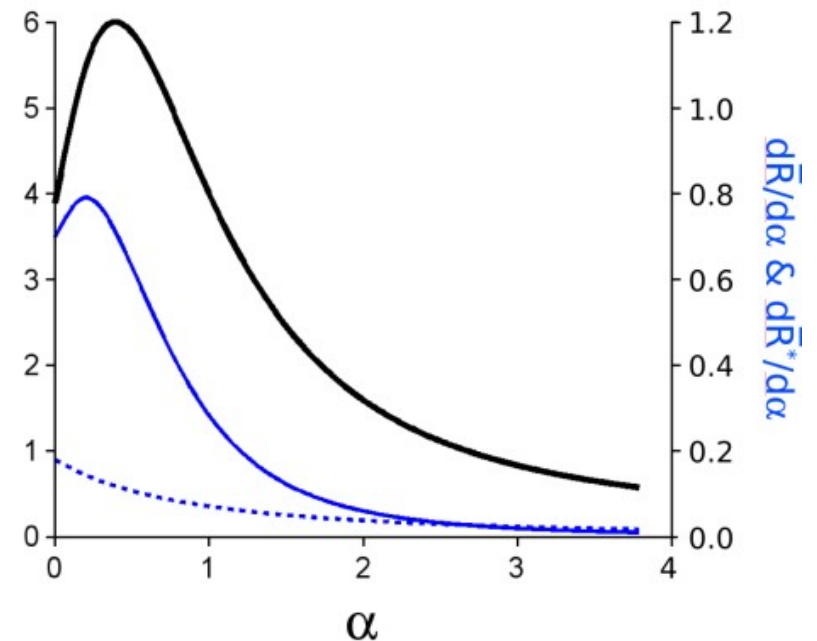
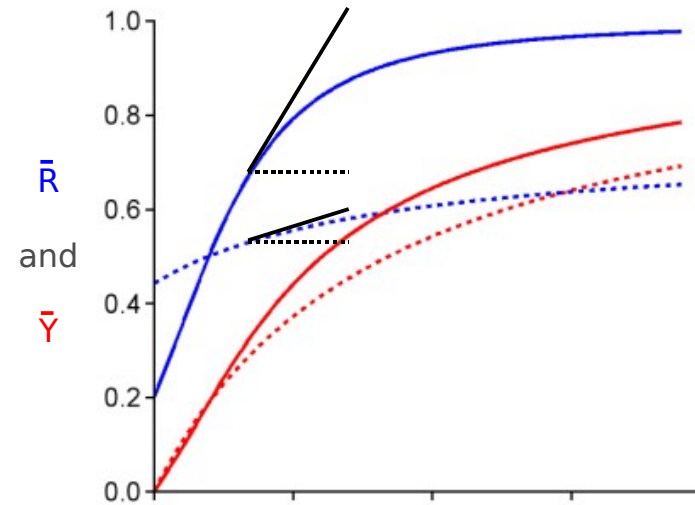
We assume that the free energy of conformational change spread over all the subunits (symmetrical protein)

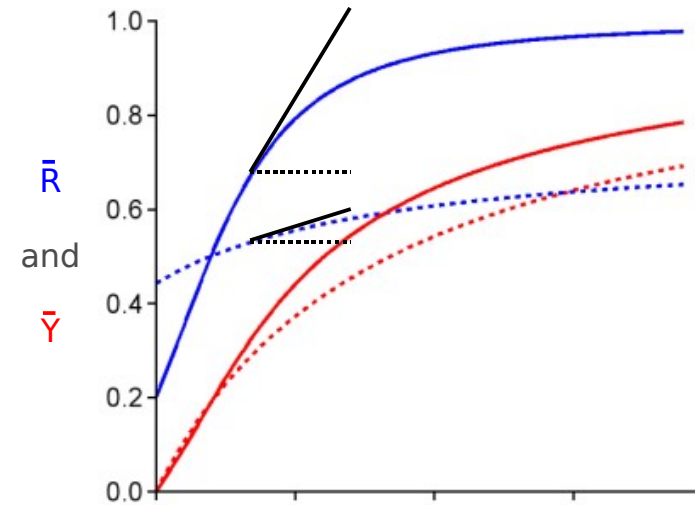
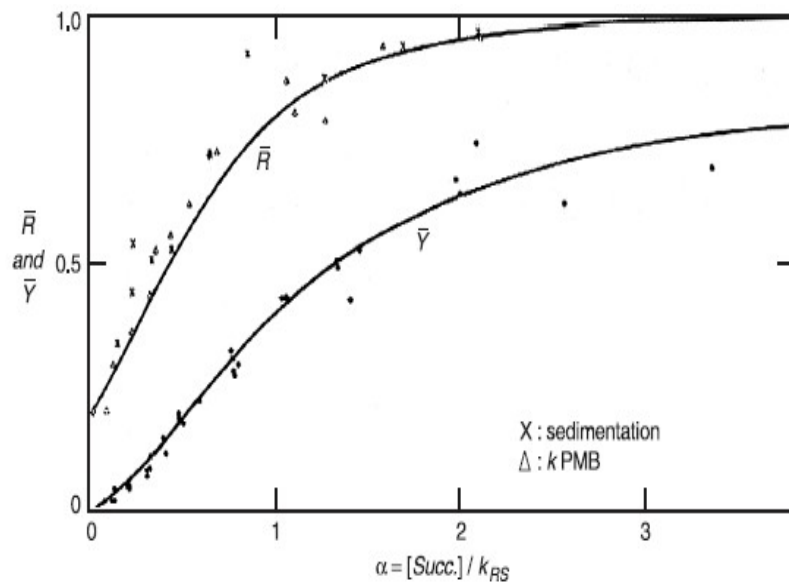
$$\lambda = \sqrt[N]{L}$$



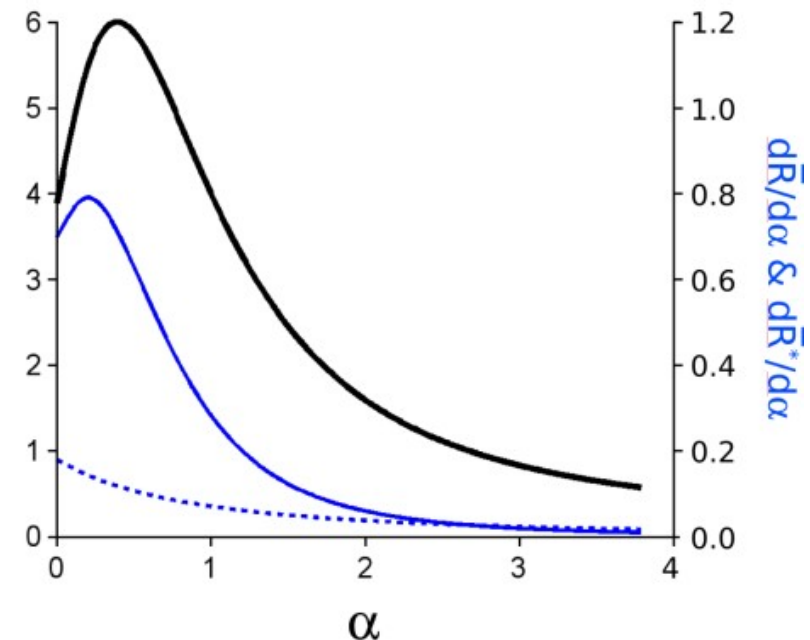


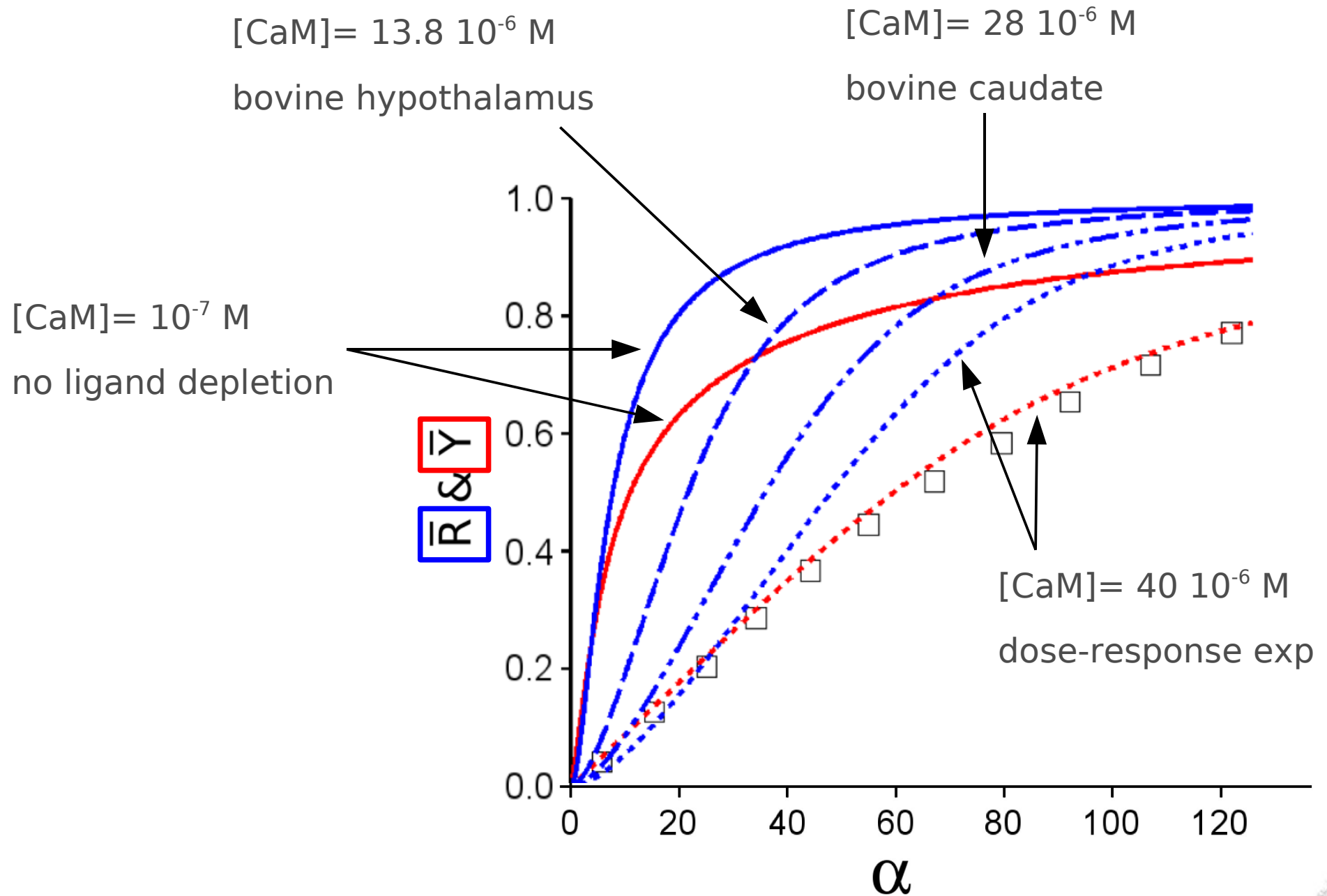
$$\nu = \frac{d\bar{R}/d\alpha}{d\bar{R}^*/d\alpha}$$

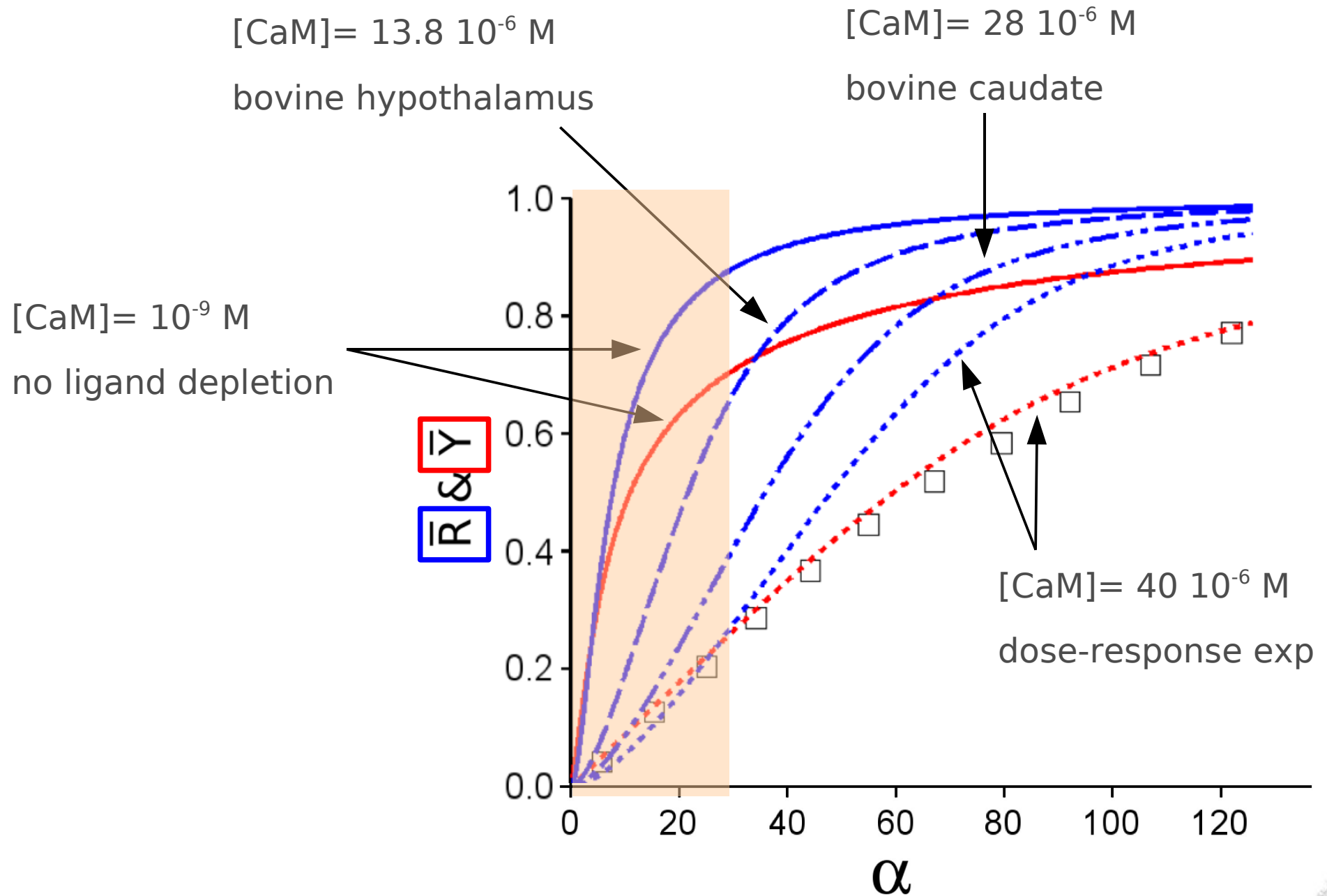


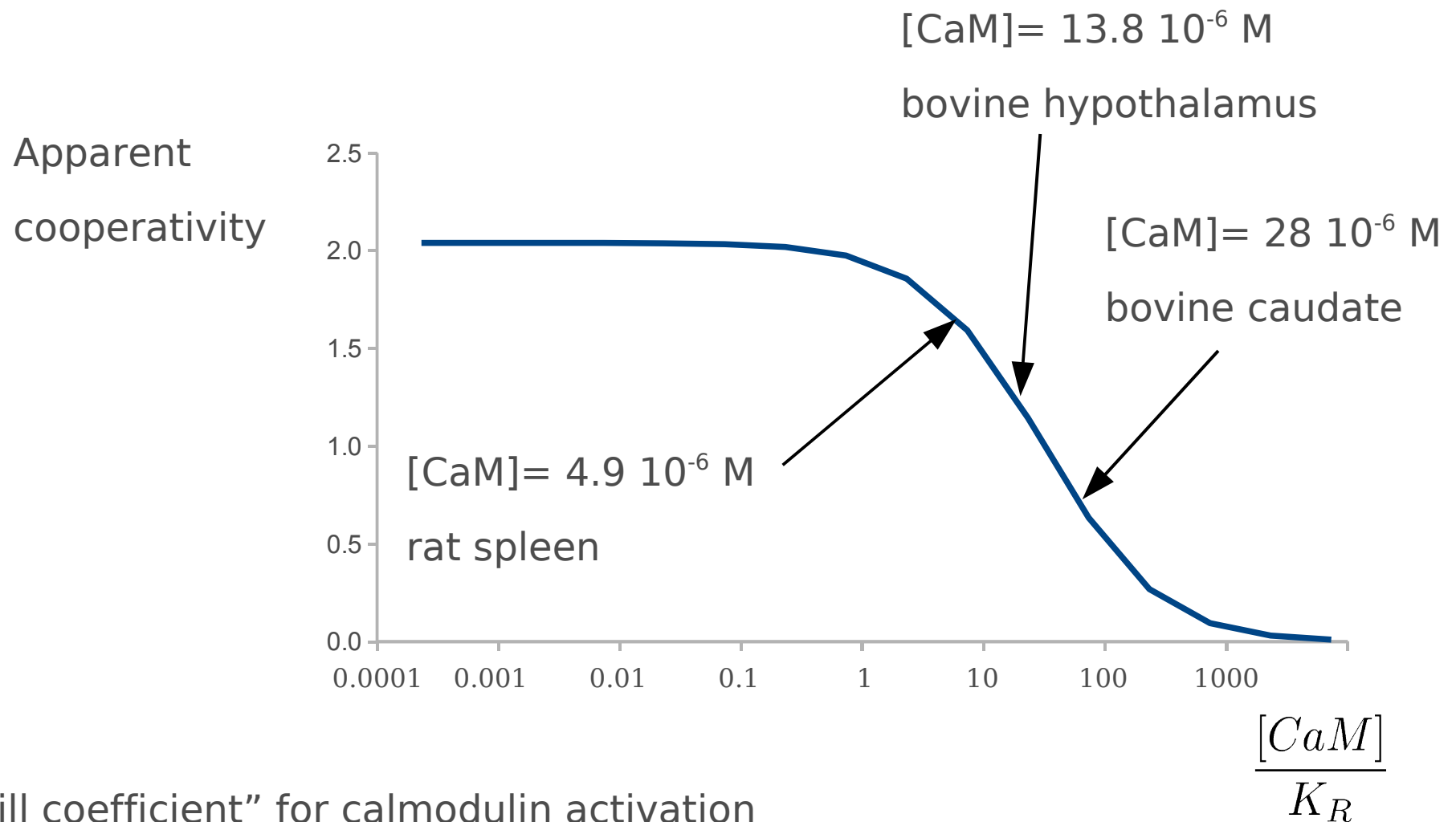


If all sites identical, the cooperativity is maximum when $\bar{R} = \bar{R}^* = 1/2$ with a value of $\nu = N$ that is the **number of binding sites**.



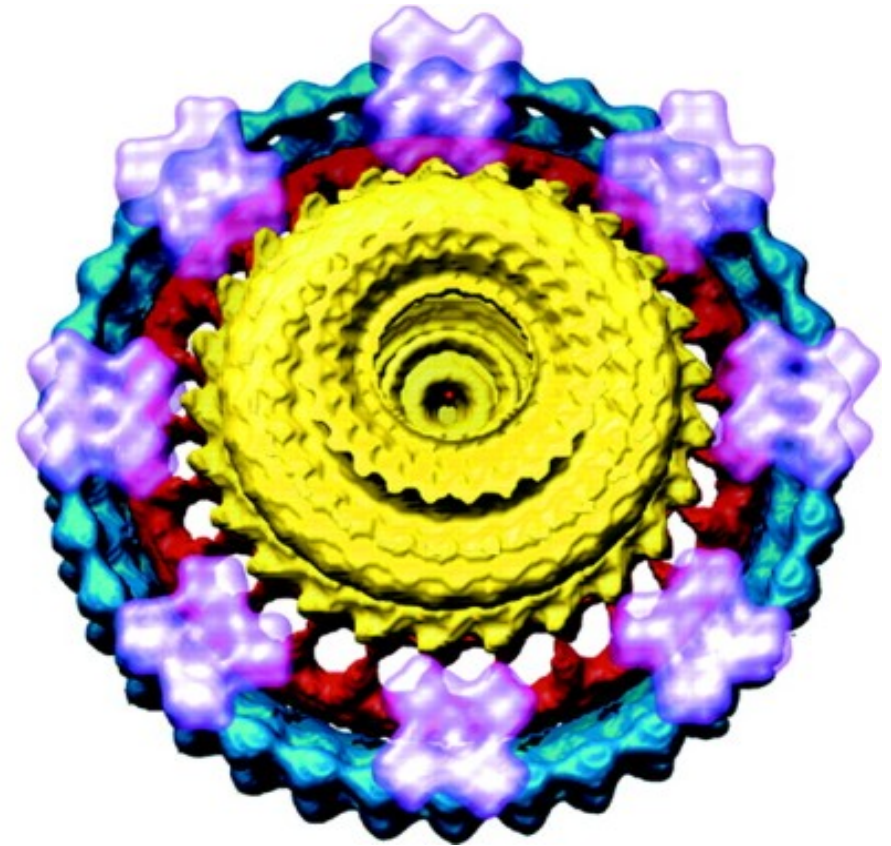
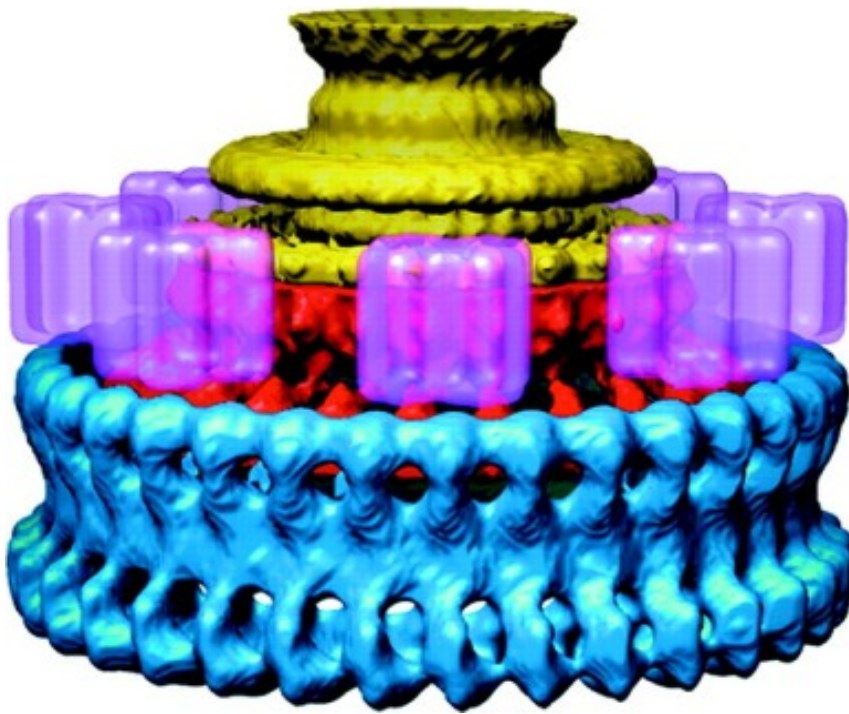


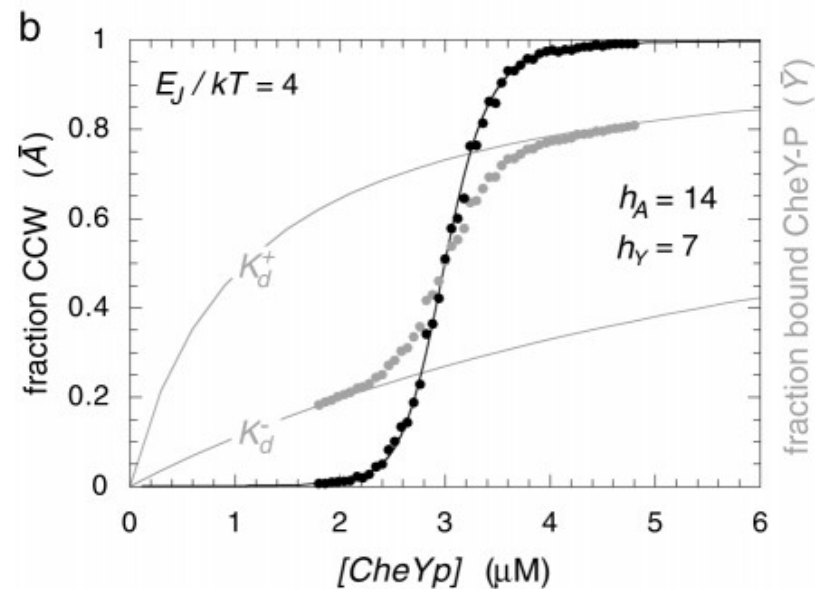
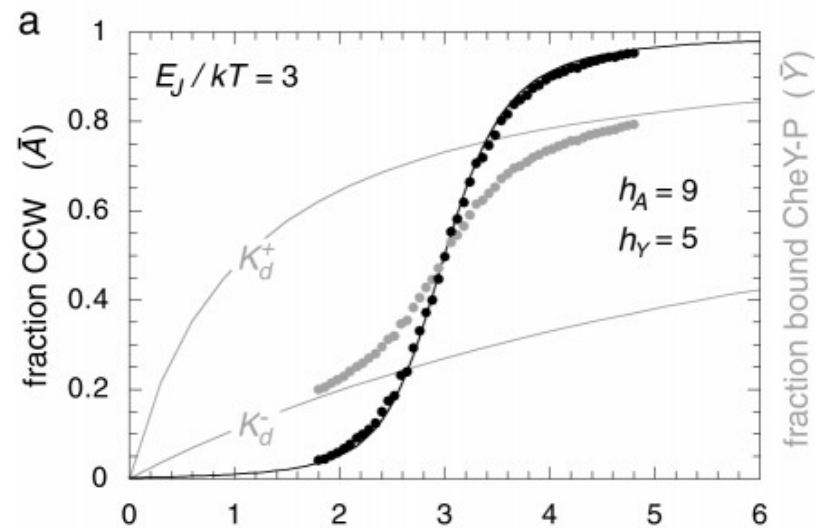
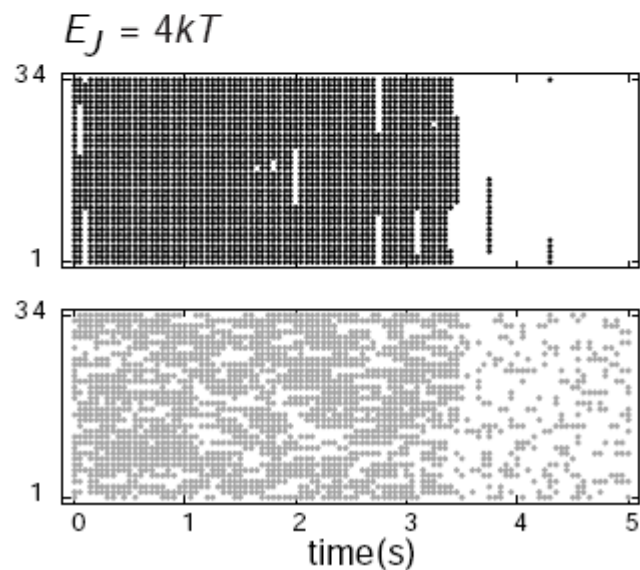
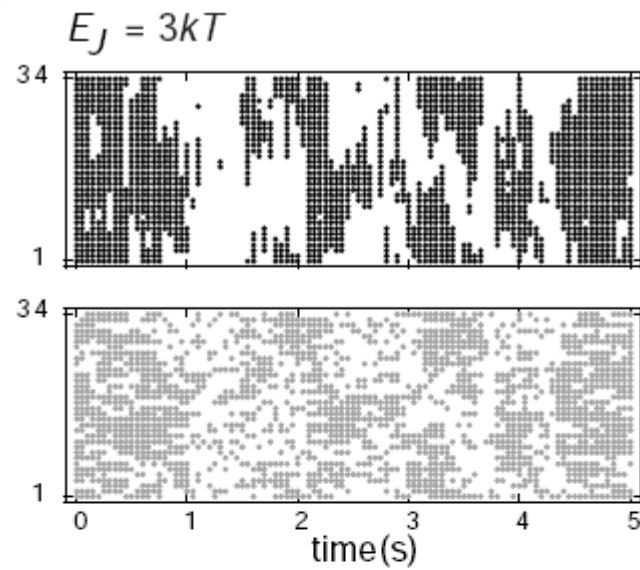




“Hill coefficient” for calmodulin activation
ranges between 1 and 1.8 ...

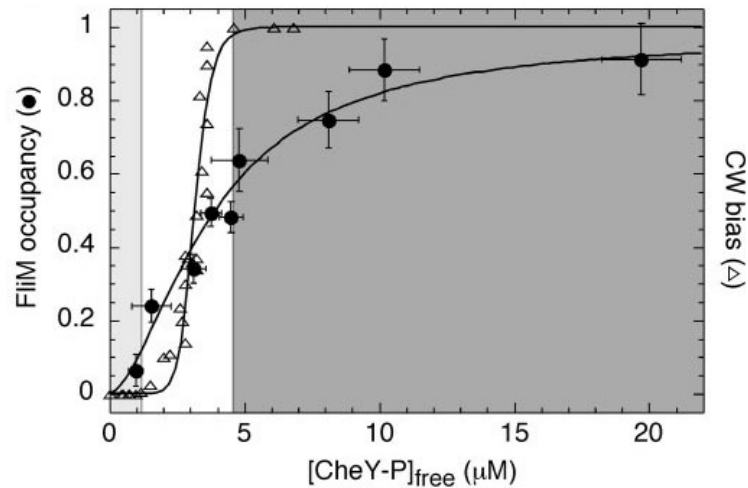






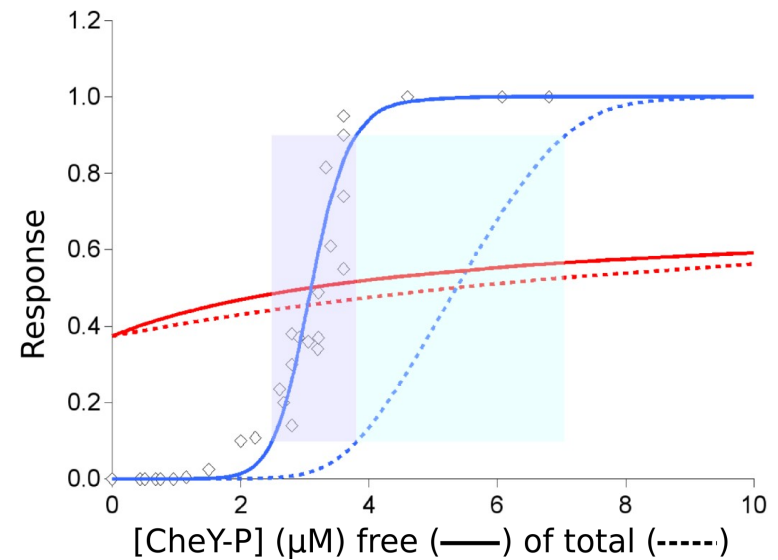
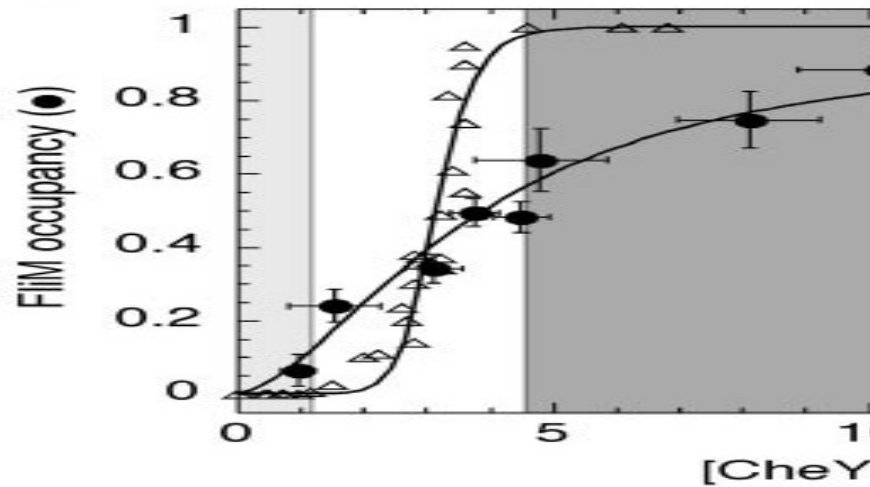
Duke, Le Novère and Bray (2001) Conformational spread in a ring of proteins: a stochastic approach to allostery. *J Mol Biol*, 308:541-553.

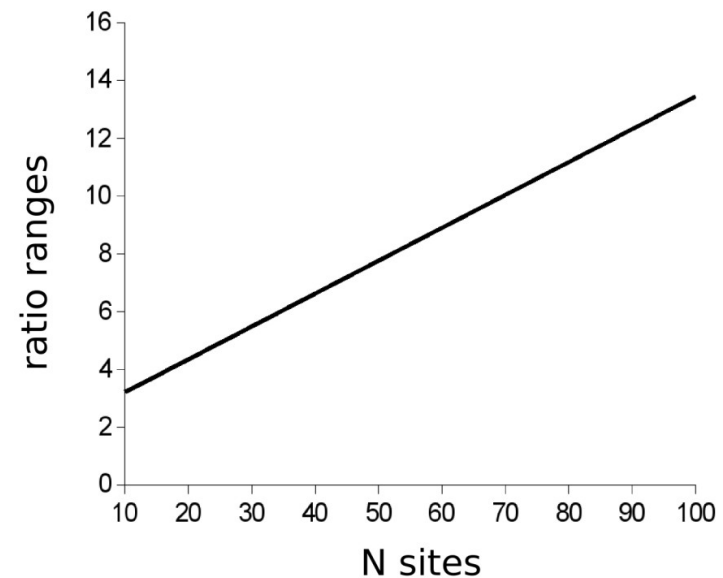
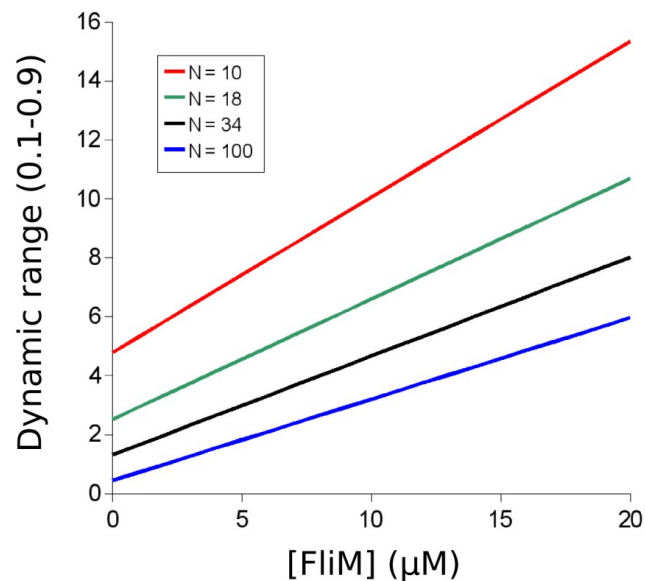
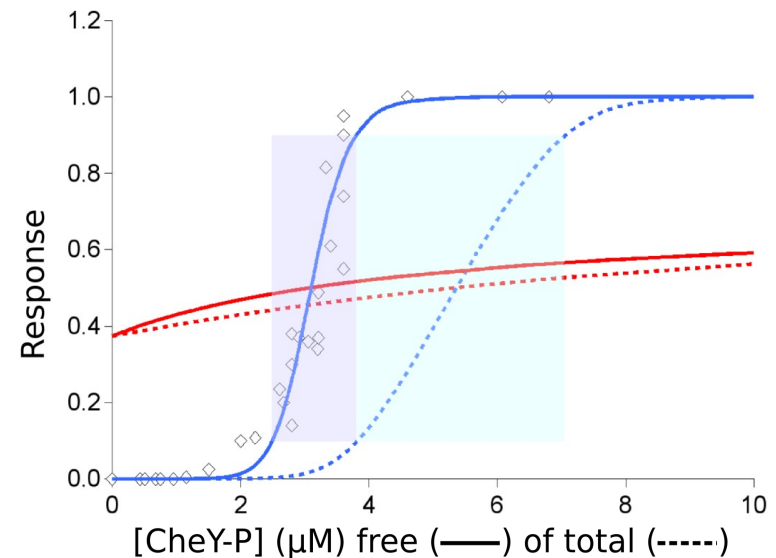
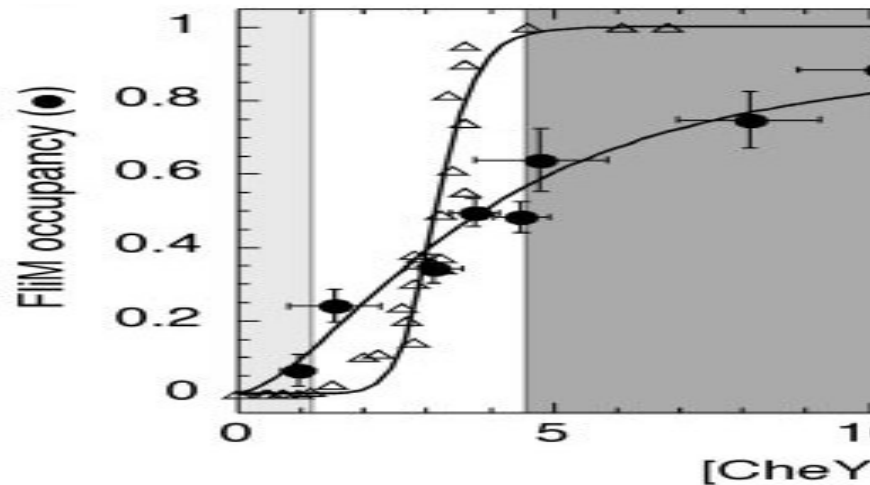


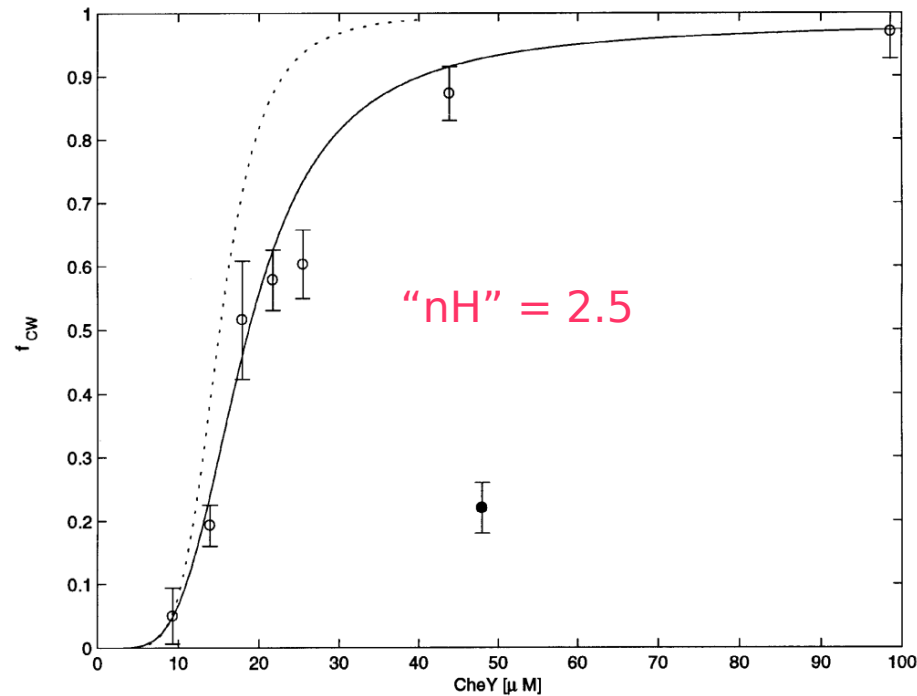


Sourjick and Berg (2002) Binding of the Escherichia coli response regulator CheY to its target measured in vivo by fluorescence resonance energy transfer. *PNAS* 99: 12669-12674

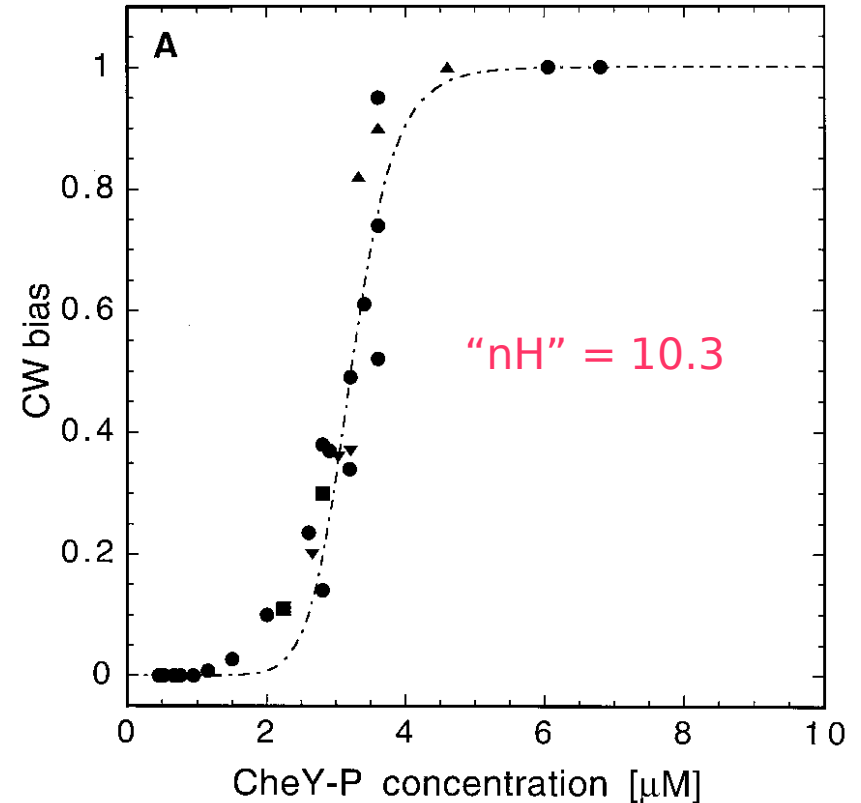








Alon et al (1998) Response regulator output in bacterial chemotaxis.
EMBO J 17: 4238-4248



Cluzel et al (2000) An ultrasensitive bacterial motor revealed by monitoring signalling proteins in single cells.
Science 287: 1652-1655



- Ligand depletion decreases the effective cooperativity



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- Modifying the concentration of the sensor may be a powerful way to quickly adapt to a new environment, and switch from a measurement mode to a detection mode.
- Generic advice for the quantitative study of cellular signalling:
 - Measure the parameters *in vitro* in controlled conditions
 - Measure quantities of interactants and subcellular location
 - Develop accurate quantitative models



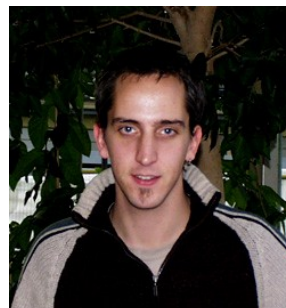


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Stuart Edelstein, CH, US

Cooperativity crew



Dominic Tolle, EI, DE



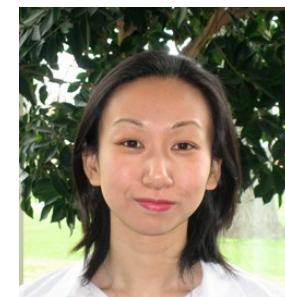
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Spatial simulation crew

BioModels crew



Lu Li, CN



Noriko Hiroi, JP

Signalling pathways crew



Viji Chelliah



Lukas Endler



Chen Li



Nicolas Rodrigues, FR



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