

1999

2003 Institut Pasteur

Student (Msc, PhD) (Jean-Pierre Changeux) Nicotinic receptors: Bioinformatics, Neuroanatomy Behaviour

CNRS Researcher (Jean-Pierre Changeux) Nicotinic receptors: **Bioinformatics** 

"A tale of two cities"

1999



2001

2003 2012 EMBL-EBI

Group leader Systems Biology Synaptic signalling: Modelling, knowledge 2012



Senior group leader Signalling neurodegeneration Stem cells

EMBO post-doc (Dennis Bray) Bacterial chemotaxis: Bioinformatics. Representation, databases Mathematical Modelling

2001

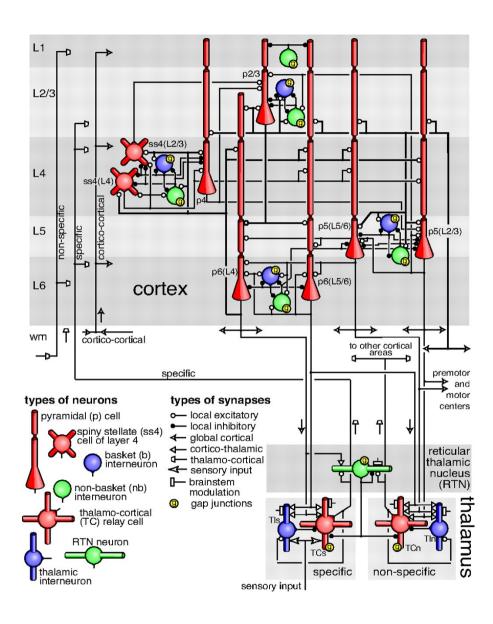


# Allosteric calcium sensors and signalling switching

Nicolas Le Novère, Babraham Institute n.lenovere@gmail.com http://lenoverelab.org



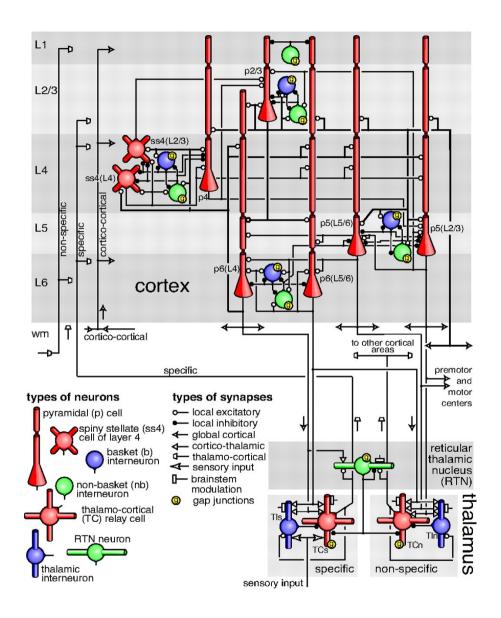
#### The brain as an electrical circuit?



Izhikevich, Edelman (2008) PNAS 105: 3593-3598



#### The brain as an electrical circuit?



**BUT** 

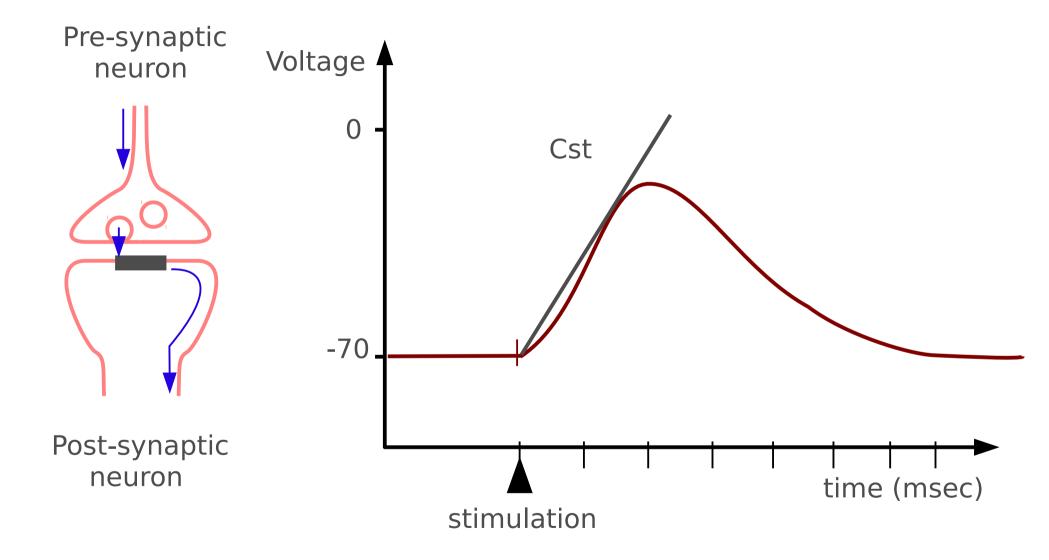
With variable topology

With variable connexions strength

Izhikevich, Edelman (2008) PNAS 105: 3593-3598



# **Excitatory post-synaptic potential**



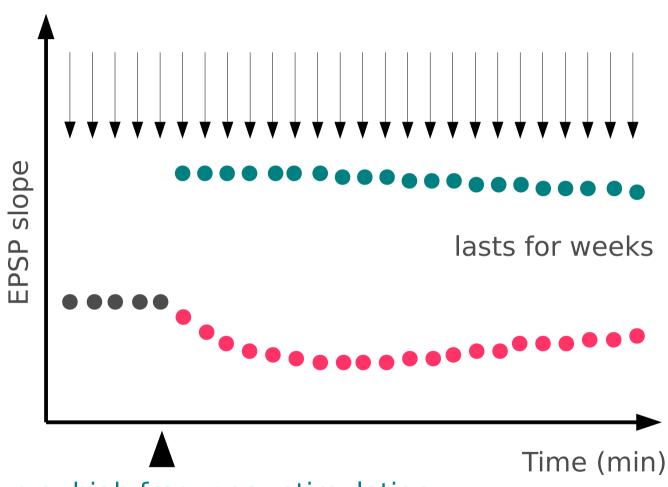


### **Bidirectional synaptic plasticity**





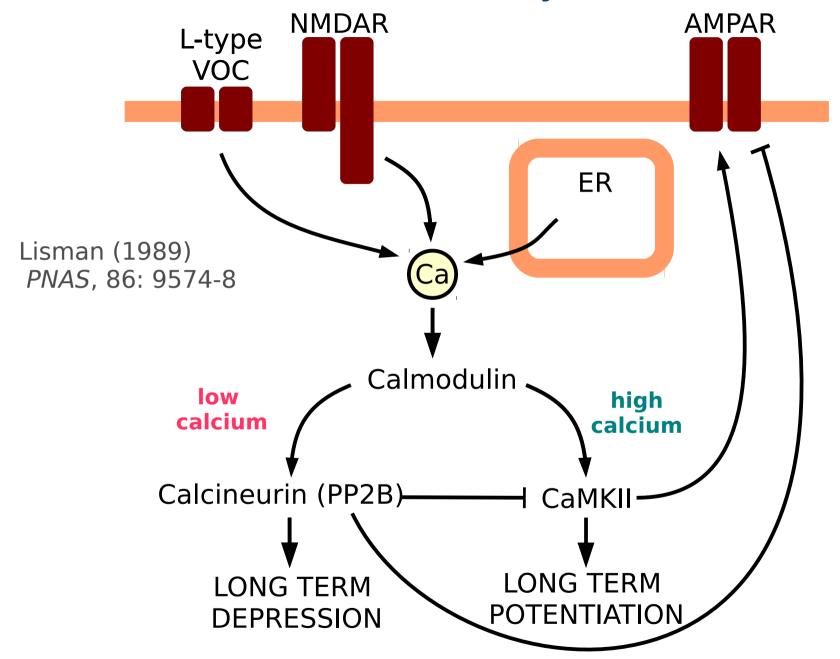
Post-synaptic neuron



e.g. high frequency stimulation e.g. low frequency stimulation

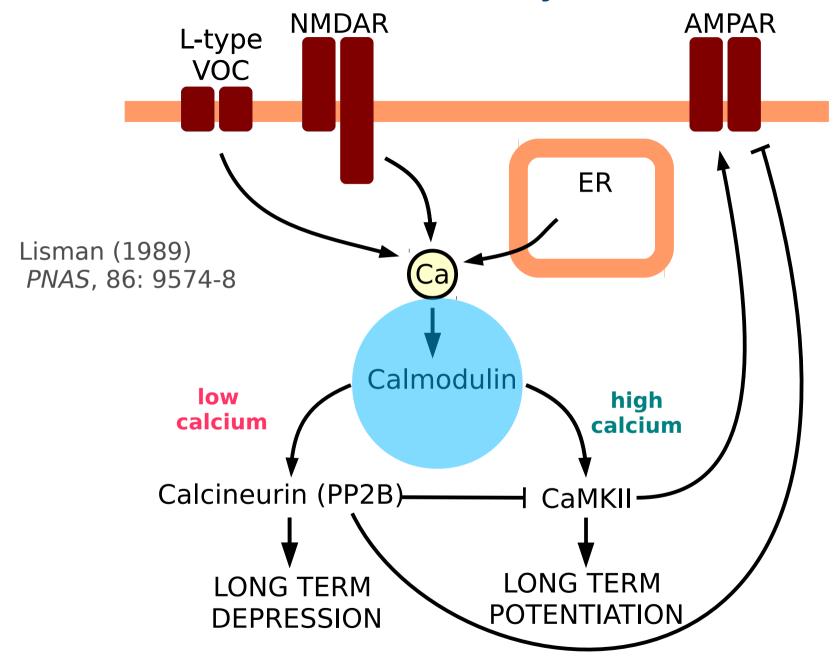


### Calmodulin, the memory switch



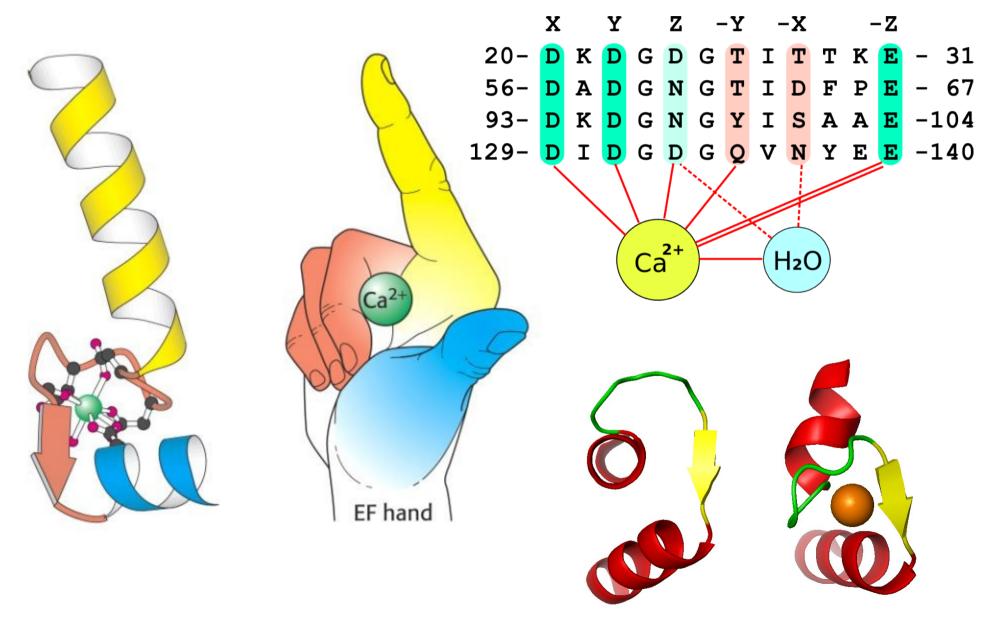


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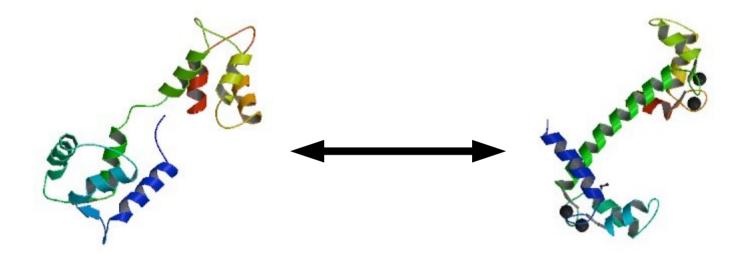




#### **Structure of Calmodulin**

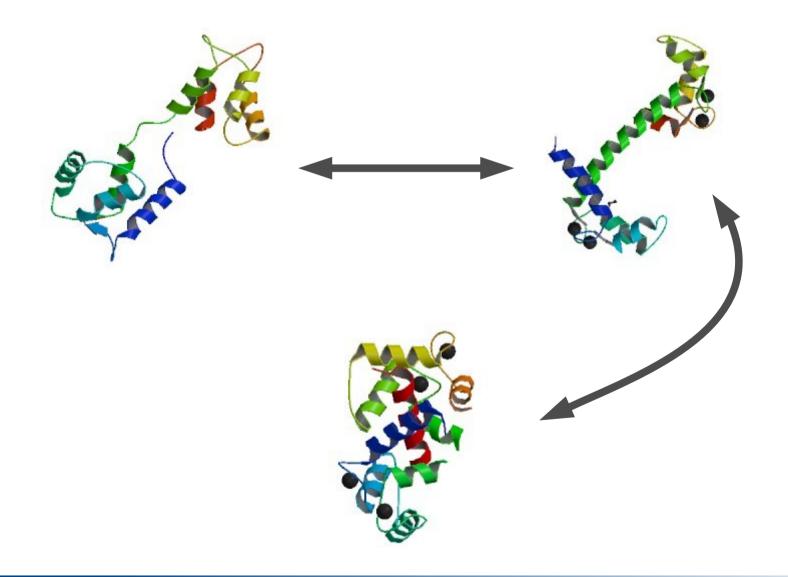


# **State transitions of calmodulin**



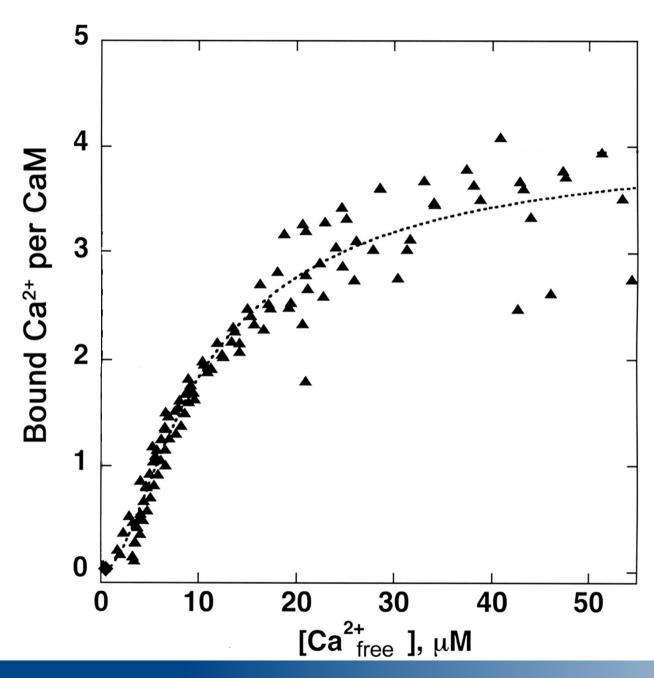


# **State transitions of calmodulin**





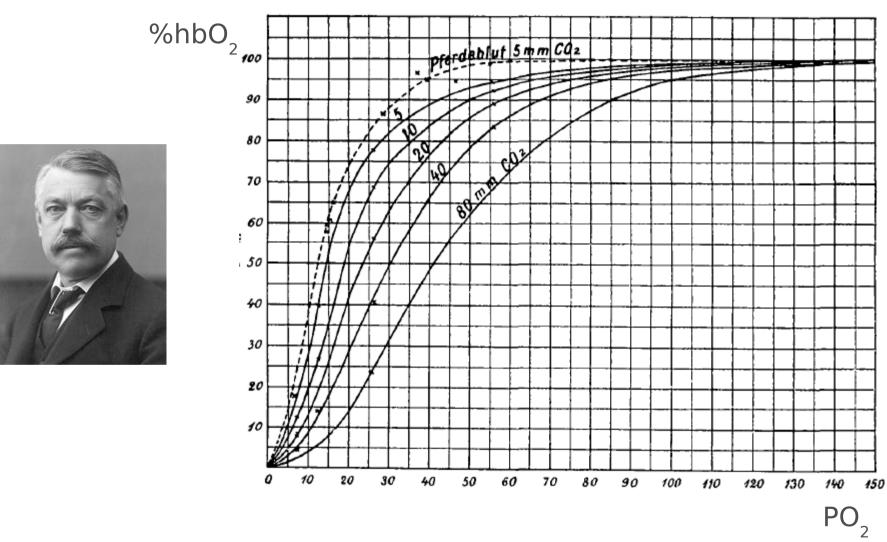
#### Calmodulin is ultra-sensitive



Shifman et al (2006) *PNAS*, 103: 13968-13973



# Origins of cooperativity: Bohr



Bohr C (1903) Theoretische behandlung der quantitativen verhältnisse bei der sauerstoff aufnahme des hämoglobins Zentralbl Physiol 17: 682



The possible effects of the aggregation of the molecules of hæmoglobin on its dissociation curves. By A. V. Hill.

In a previous communication Barcroft and I gave evidence which seemed to us to prove conclusively that dialysed hæmoglobin consists simply of molecules containing each one atom of iron. The molecular weight is therefore Hb=16,660. These experiments have not been published yet, but I shall assume the results.

Other observers (Reid, Roaf, Hüfner and Gansser) working on different solutions have obtained divergent results. The method used by all of them was the direct estimation of the osmotic pressure, by means of a membrane permeable to salts, but not to hæmoglobin. The method involves a relatively large error, because the quantity measured is small. It is doubtful however whether this can explain the discordant results.

Our work led me to believe that the divergence between the results of different observers was due to an aggregation of the hæmoglobin molecules by the salts present in the solution, a consequent lowering of the number of molecules, and an increase in the average molecular weight as observed by the osmotic pressure method. To test this hypothesis I have applied it to several of the dissociation curves obtained by Barcroft and Camis with hæmoglobin in solutions of various salts, and with hæmoglobin prepared by Bohr's method.

The equation for the reaction would be

$$Hb + O_2 \rightleftharpoons HbO_2$$
,  
 $Hb_n + nO_2 \rightleftharpoons Hb_nO_{2n}$ ,

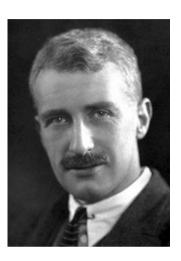
where  $Hb_n$  represents the aggregate of n molecules of Hb. I have supposed that in every solution there are many different sized aggregates, corresponding to many values of n.

If there were in the solution only Hb and Hb<sub>2</sub> the dissociation curve would be

$$y = \lambda \frac{K'x^2}{1 + K'x^2} + (100 - \lambda) \frac{Kx}{1 + Kx}$$
 .....(A),

where  $\lambda^{\circ}/_{0}$  is as  $\mathrm{Hb}_{2}$ ,  $(100 - \lambda)^{\circ}/_{0}$  as  $\mathrm{Hb}$ , K' is the equilibrium constant of the reaction  $\mathrm{Hb}_{2} + 2\mathrm{O}_{2} \rightleftharpoons \mathrm{Hb}_{2}\mathrm{O}_{4}$  and K that of  $\mathrm{Hb} + \mathrm{O}_{2} \rightleftharpoons \mathrm{Hb}\mathrm{O}_{2}$ : K has the value 125 (Barcroft and Roberts).

Hill (1910) J Physiol 40: iv-vii.





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Hill (1910) J Physiol 40: iv-vii.

Now it is unlikely that in either of these cases there is only Hb and Hb<sub>2</sub>: and as the calculation of the constants in these equations is very tedious I decided to try whether the equation

$$y = 100 \frac{Kx^n}{1 + Kx^n}$$
 .....(B)

would satisfy the observations.



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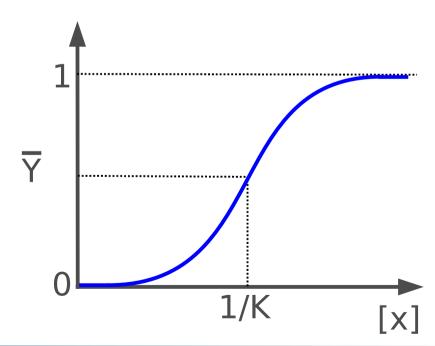
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### Hill equation can be linearised

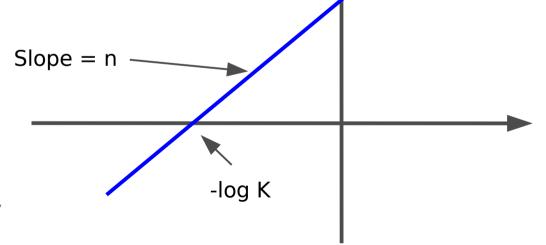
$$\bar{Y} = \frac{K^{\mathsf{n}}[X]^{\mathsf{n}}}{1 + K^{\mathsf{n}}[X]^{\mathsf{n}}}$$

Hill equation

$$\log \frac{\bar{Y}}{1 - \bar{Y}} = n \log K + n \log[x] \quad \text{Hill plot}$$

Effect increases in function of the signal to the power of n: n>1, ultra-sensitive n<1, infra-sensitive

BUT cooperativity of ligand, not of binding sites: unique affinity





# Origins of cooperativity: Adair-Klotz

#### THE HEMOGLOBIN SYSTEM.

#### VI. THE OXYGEN DISSOCIATION CURVE OF HEMOGLOI

By G. S. ADAIR.

WITH THE COLLABORATION OF A. V. BOCK AND H. FIELD, J. (From the Medical Laboratories of the Massachusetts General Hos Boston.)

(Received for publication, January 7, 1925.)

This work gives the oxygen dissociation curves of so previously investigated in regard to their acid-binding and

Adair (1925) J Biol Chem 63: 529

$$\bar{Y} = \frac{1}{n} \frac{K_1[x] + 2K_2[x]^2 + 3K_3[x]^3 + 4K_4[x]^4}{1 + K_1[x] + K_2[x]^2 + K_3[x]^3 + K_4[x]^4}$$

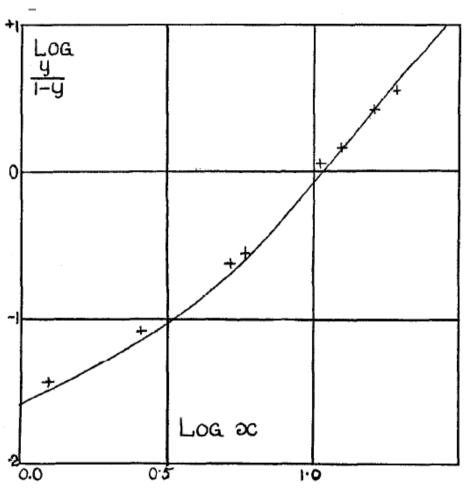


Fig. 2. Test of formula (6). Curve drawn from 6 experimental points from Table IV.



# Origins of cooperativity: Adair-Klotz

#### THE HEMOGLOBIN SYSTEM.

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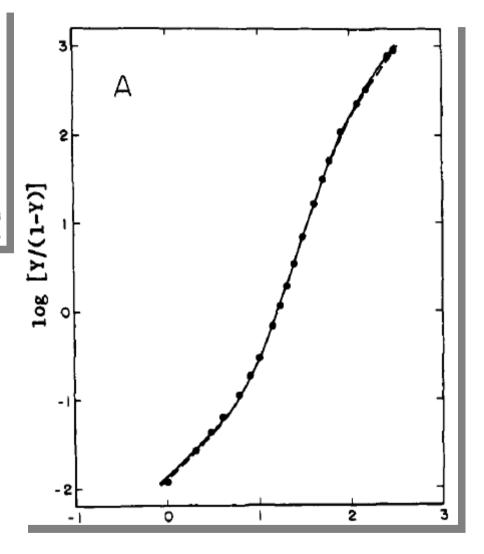
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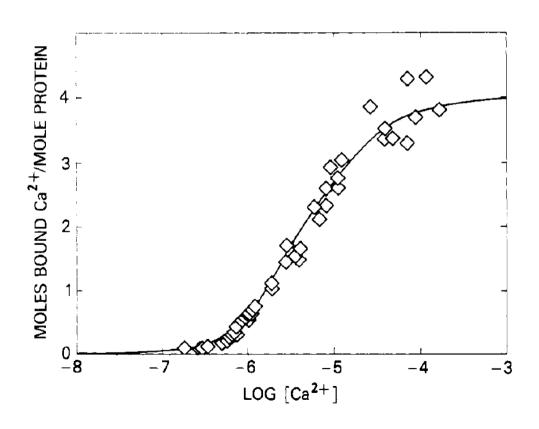
Imai (1973) Biochemistry 12: 798-808



### Adair-Klotz model applied to Calmodulin

Klotz (1946) The Application of the Law of Mass Action to Binding by Proteins. Interactions with Calcium. *Arch Biochem*, 9:109–117.

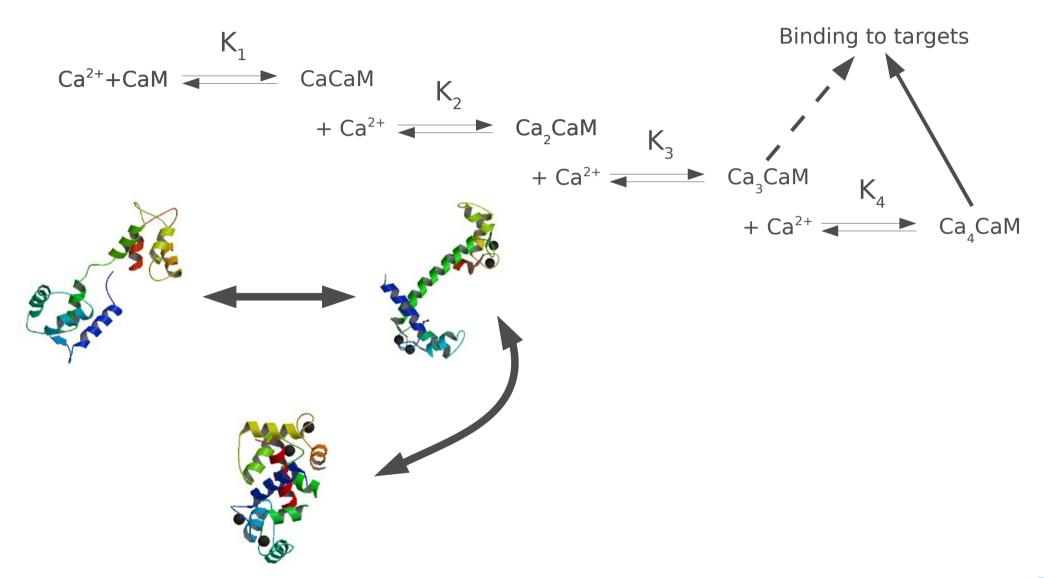
$$\bar{Y} = \frac{1}{n} \frac{K_1[Ca] + 2K_1K_2[Ca]^2 + 3K_1K_2K_3[Ca]^3 + 4K_1K_2K_3K_4[Ca]^4}{1 + K_1[Ca] + K_1K_2[Ca]^2 + K_1K_2K_3[Ca]^3 + K_1K_2K_3K_4[Ca]^4}$$



Crouch and Klee (1980) Biochemistry, 19: 3692-3698

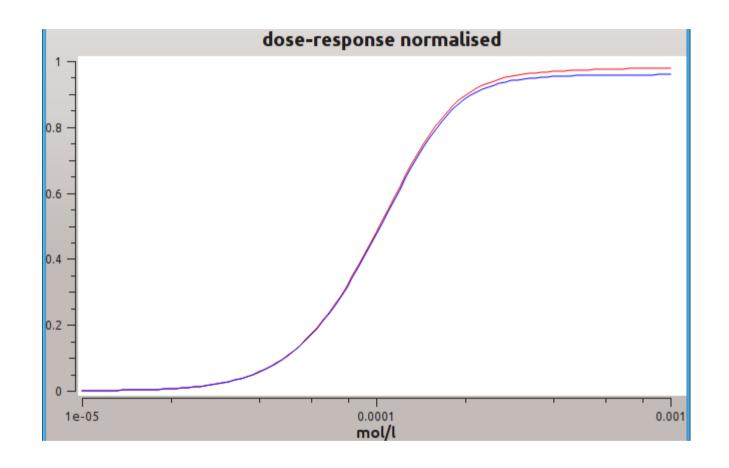


# **Corresponding induced-fit model**





#### That does not work ...



 $[\textbf{CaN}] = [\textbf{CamKII}] = [\textbf{CaM}]/10 \; ; \; \textbf{Kd\_CaMKII} = 10 \times \textbf{Kd\_CaN}; \; \textbf{Software COPASI}$ 



#### We knew it would not work

- Calmodulin bound to 3 calcium ions can activate calcineurin
  - Kincaid and Vaughan (1986). PNAS, 83: 1193-1197
- Calmodulin bound to 2 calcium ions can bind CaMKII
  - Shifman et al (2006). *PNAS*, 103: 13968-13973
- Calmodulin affinity for calcium increases once bound to CaMKII
  - Shifman et al (2006) [but many previous reports on other targets: e.g. Burger et al (1983). *JBC*, 258: 14733-14739;
     Olwin et (1984). *JBC* 259: 10949-10955]
- Calcium activates both LTP and LTD through calmodulin
  - Lisman (1989) *PNAS*, 86: 9574-9578
  - High  $[Ca^{2+}]$  (high freq)  $\cong$  CaMKII; Low  $[Ca^{2+}]$  (low freq)  $\cong$  Calcineurin



### Allostery and state selection

Monod, Wyman, Changeux (1965). On the nature of allosteric transitions: a plausible model.
 J Mol Biol, 12: 88-118

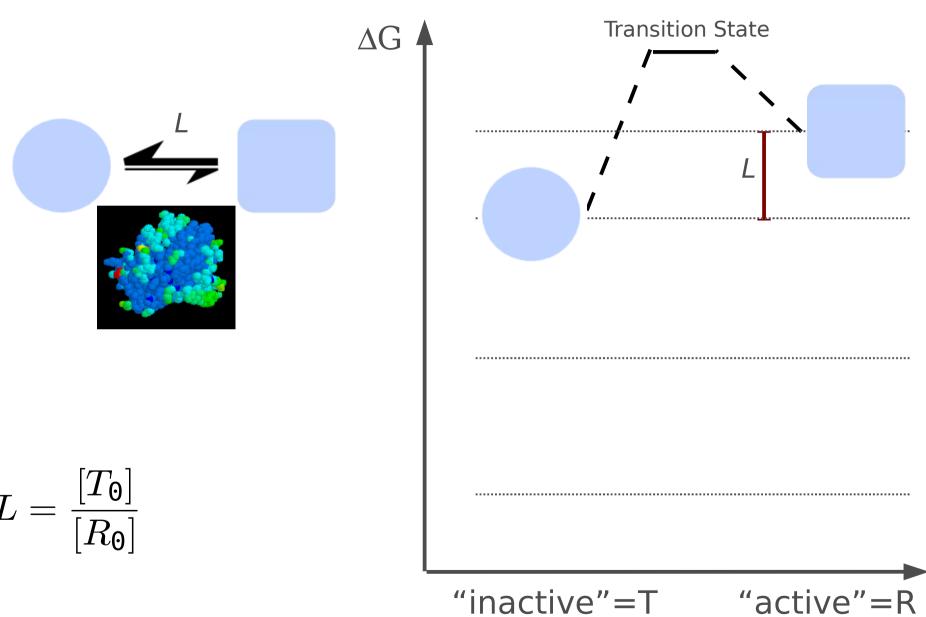




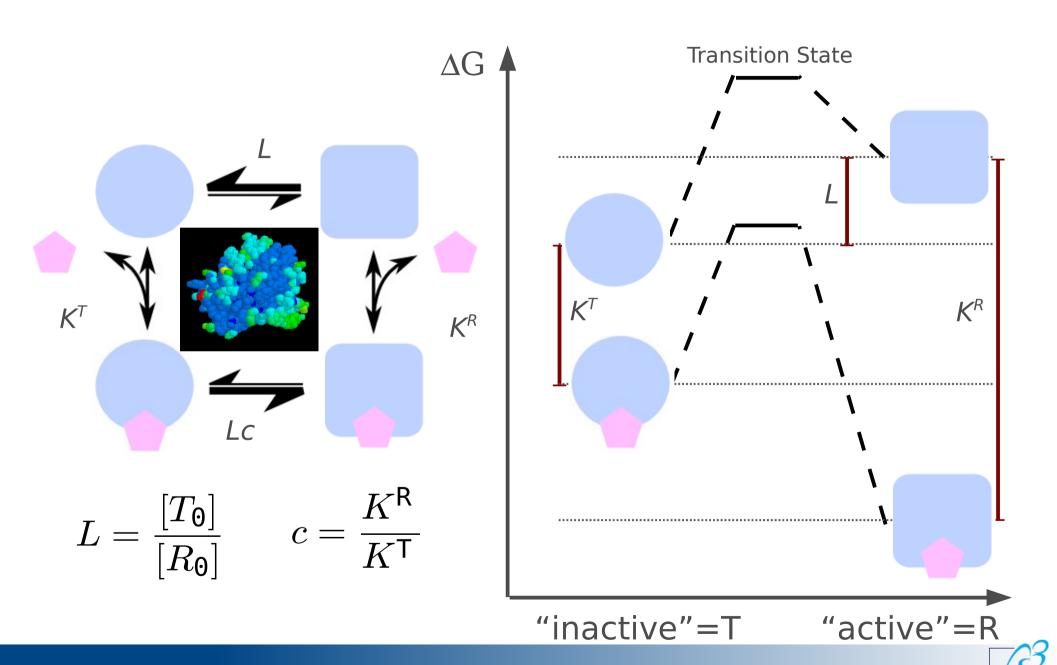




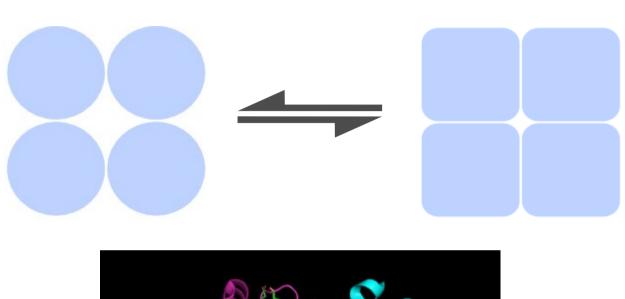
# Modulation of thermal equilibria ≠ induced-fit

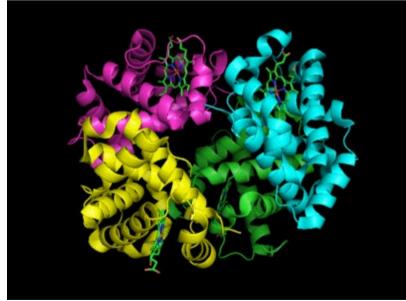


# Modulation of thermal equilibria ≠ induced-fit



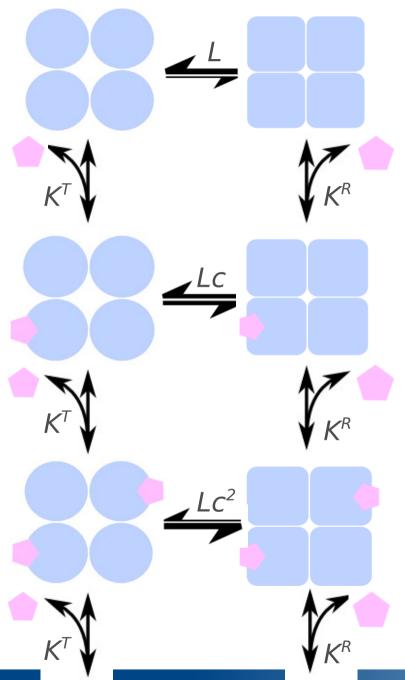
# **Concerted transitions** ≠ sequential model





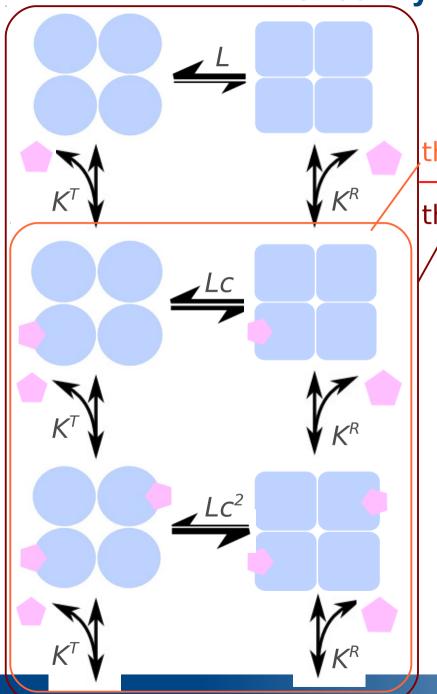


# Monod-Wyman-Changeux model



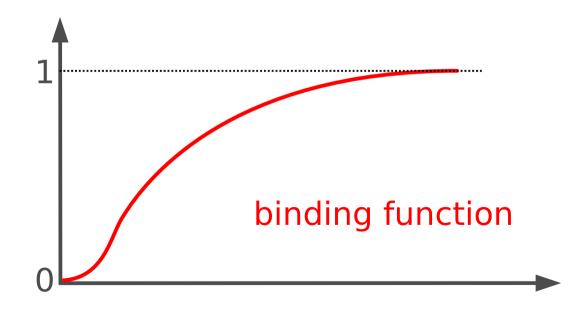


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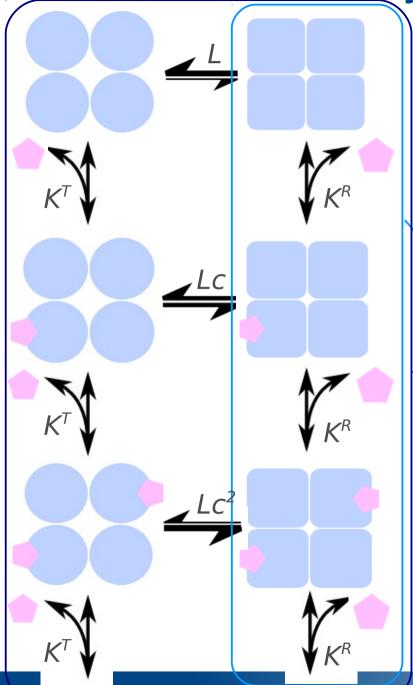


$$\alpha = \frac{[x]}{K^{\mathsf{R}}}$$

$$\overline{\overline{Y}} = \overline{Y} = \frac{\alpha(1+\alpha)^{\text{n i } 1} + Lc\alpha(1+c\alpha)^{\text{n i } 1}}{(1+\alpha)^{\text{n}} + L(1+c\alpha)^{\text{n}}}$$

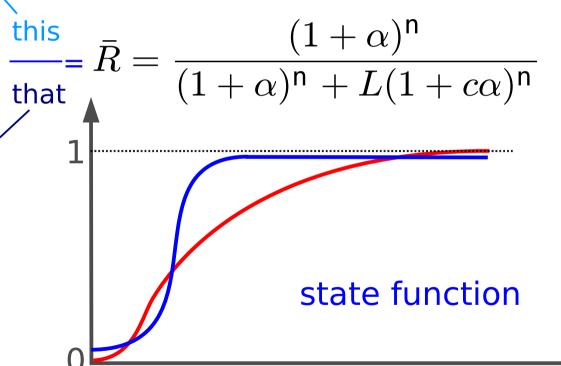


### **Monod-Wyman-Changeux model**



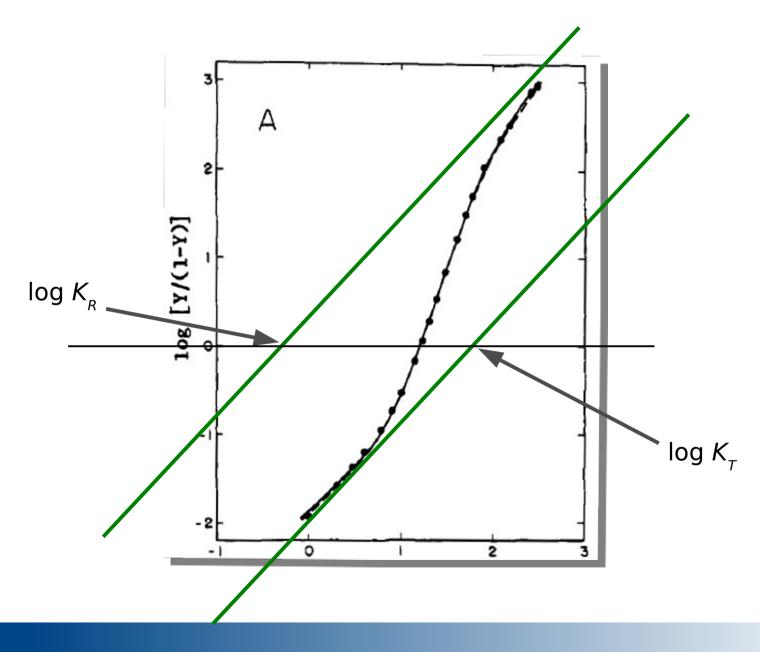
$$\alpha = \frac{[x]}{K^{\mathsf{R}}}$$

$$\bar{Y} = \frac{\alpha(1+\alpha)^{\operatorname{ni} 1} + Lc\alpha(1+c\alpha)^{\operatorname{ni} 1}}{(1+\alpha)^{\operatorname{n}} + L(1+c\alpha)^{\operatorname{n}}}$$



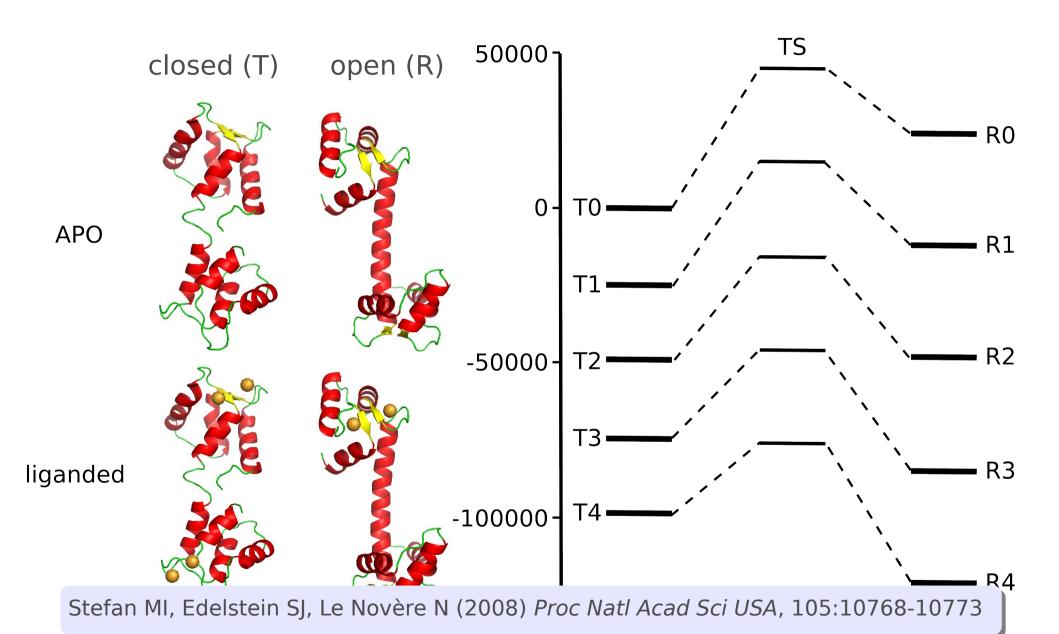


# "Hill" Plot for MWC model

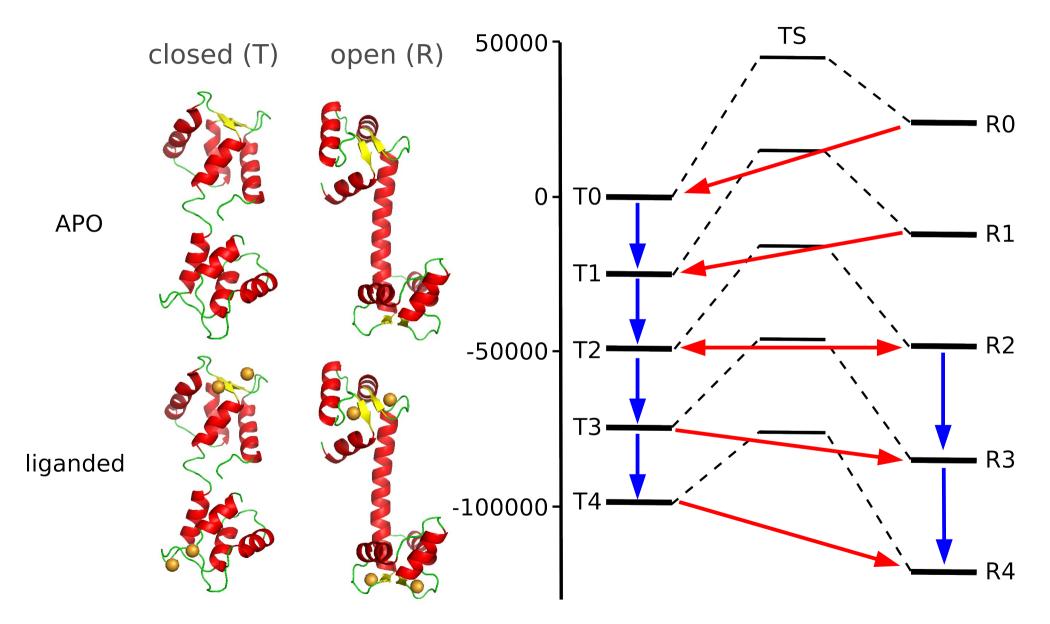




#### Allosteric model of Calmodulin activation

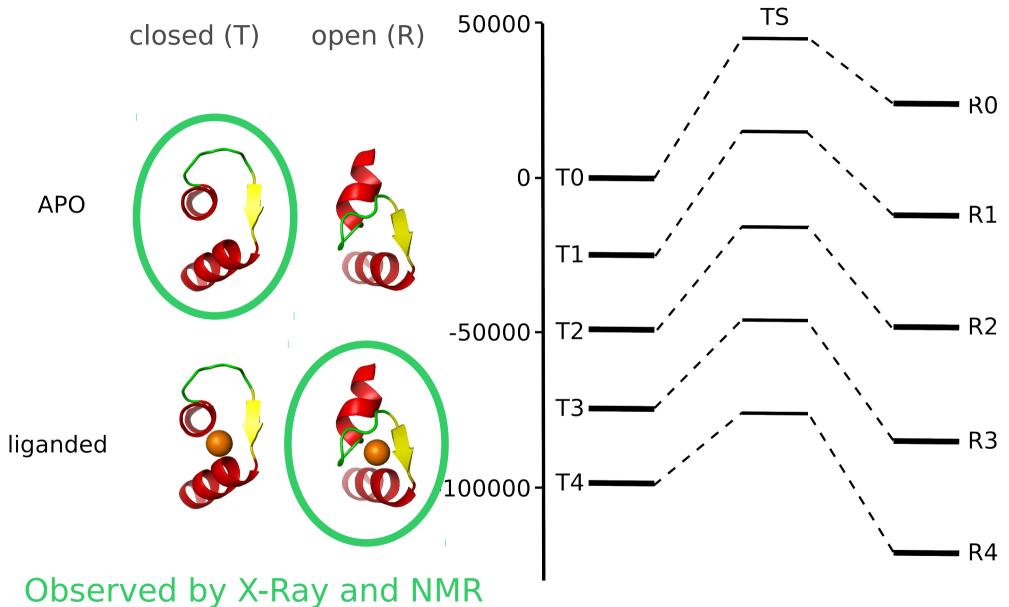


#### **Allosteric model of Calmodulin activation**



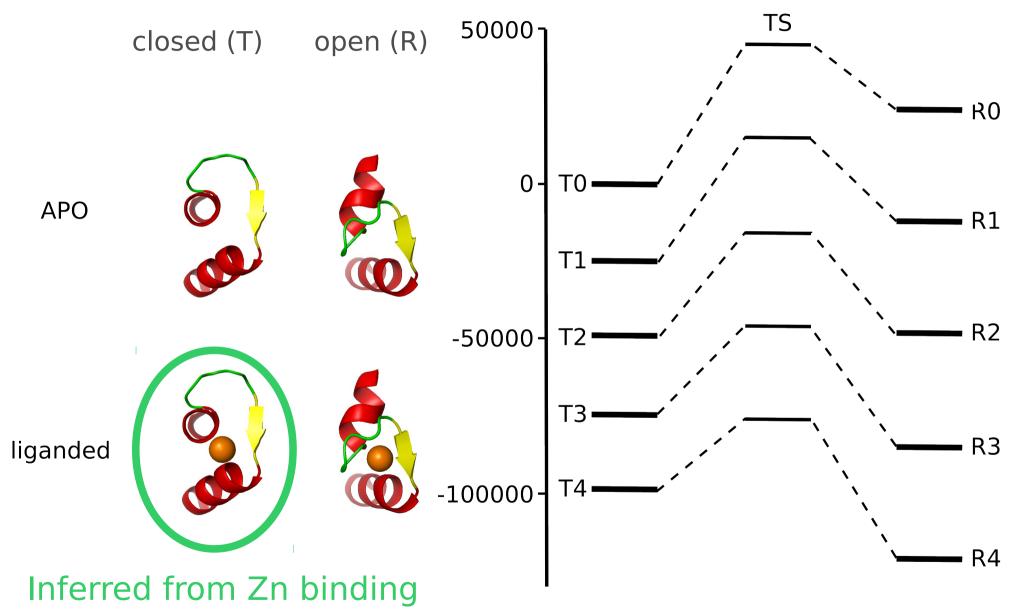


#### **Observation Vs. Prediction**

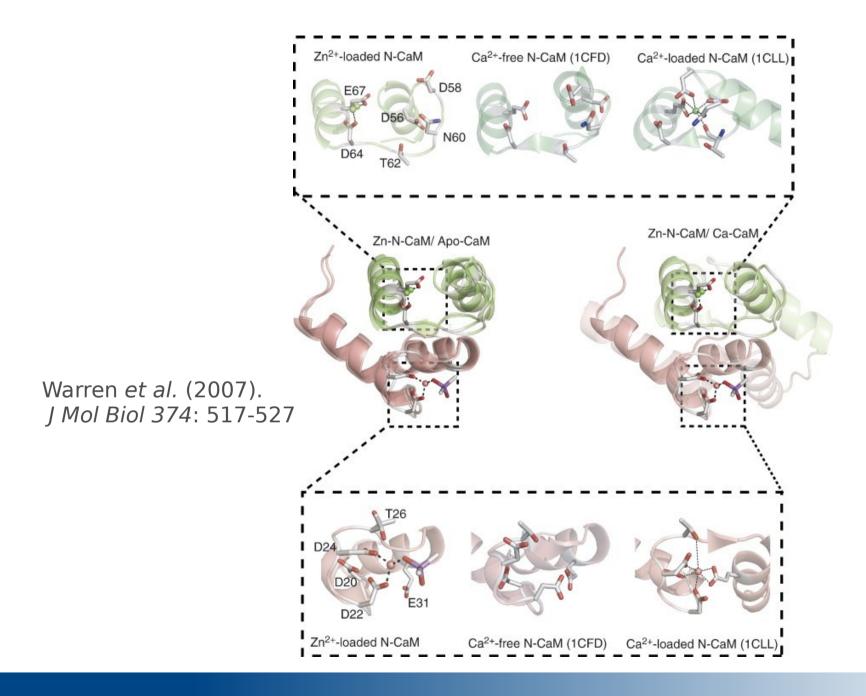




#### **Observation Vs. Prediction**

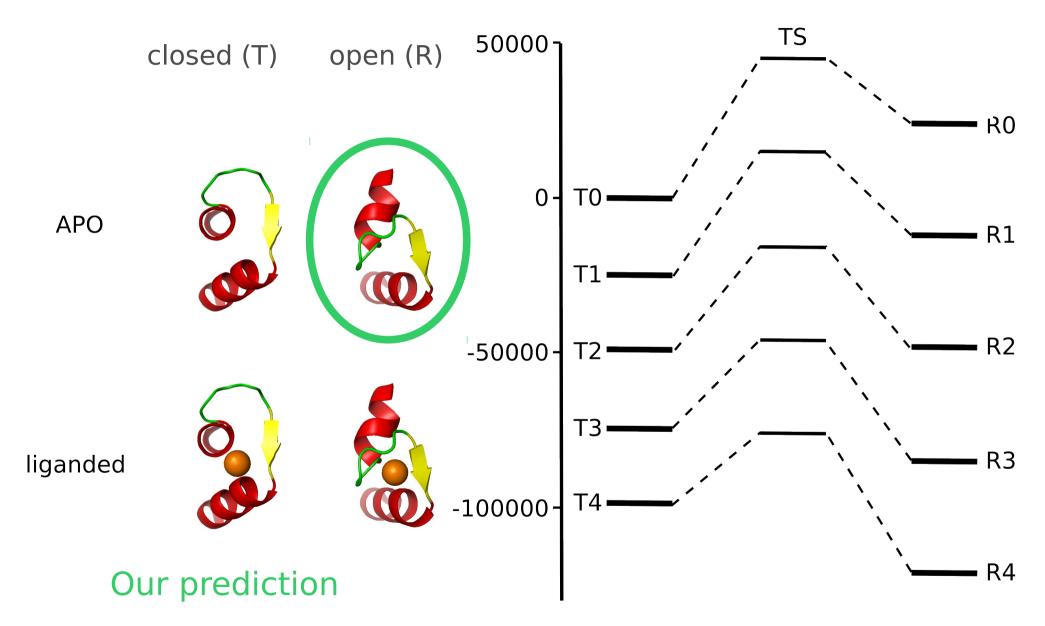








### **Observation Vs. Prediction**





## **Procedure for parameter estimations**

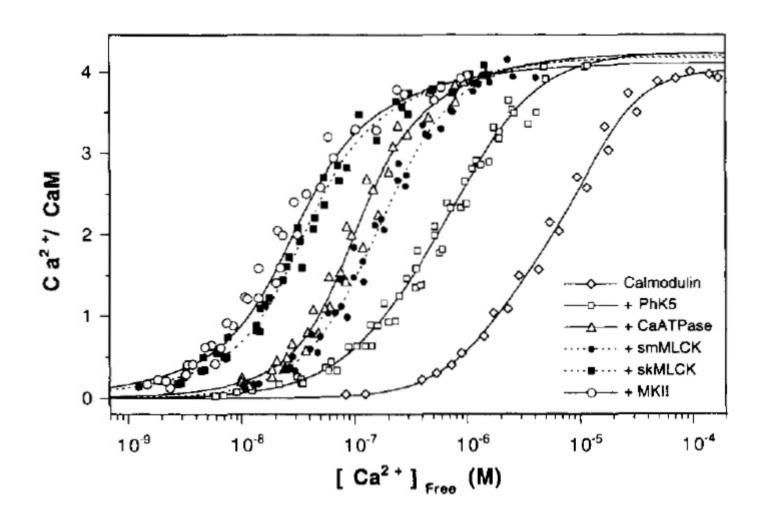
 Extension of the MWC model to support any number of sites, with different affinities, per "protomer"; several protomers per subunit

Stefan M.I., Edelstein S.J., Le Novère N. BMC Systems Biology (2009), 3: 68

Hypothesis: homogeneous distribution of transition energy = unique c. Estimation of L and c based on Ca<sup>2+</sup> binding in presence of targets: none, skMLCK, PhK5, CaATPase (Peersen et al (1997) Prot Sci 6: 794-807). 100 000 parameter sets. 13 identical minima.



## **Targets as allosteric effectors**



Peersen et al. (1997) Prot Sci, 6: 794-807



## **Procedure for parameter estimations**

 Extension of the MWC model to support any number of sites with different affinities per "protomer" and several protomers per subunit

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- Estimation of affinities for the four sites using calcium dissociation constants for Complete CaM (Bayley et al (1996) *Prot Sci* 5: 1215-1228), N and C term Mutants (Shifman et al (2006) *PNAS*, 103: 13968-13973), R-only skMLCK (Peersen et al (1997) Prot Sci 6: 794-807). 25 millions parameter sets.



### 1 in 20000 active w/o Ca<sup>2+</sup>



$$C=3.96\ 10^{-3}$$

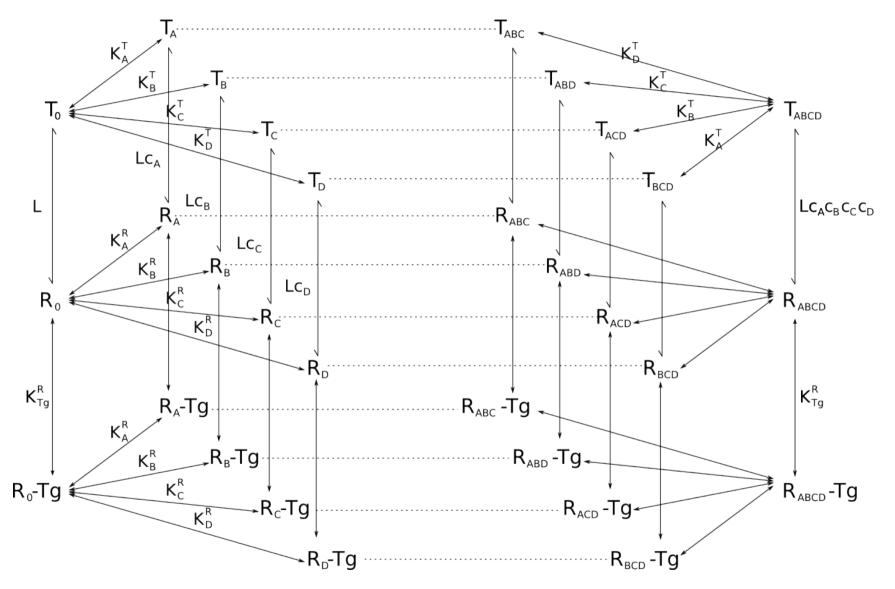
Affinity of Ca<sup>2+</sup> for "open state" 250 times higher than for "closed state"

$$K_{A}^{R} = 8.32 \ 10^{-6}$$
  
 $K_{B}^{R} = 1.66 \ 10^{-8}$   
 $K_{C}^{R} = 1.74 \ 10^{-5}$   
 $K_{D}^{R} = 1.45 \ 10^{-8}$ 

2 high, 2 low, as anticipated



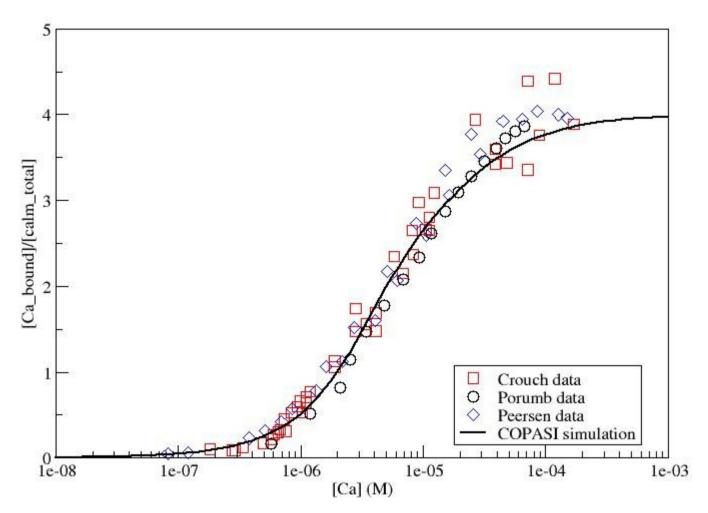
## Full mechanistic thermodynamic model



320 reactions



## **Comparison with experiments**



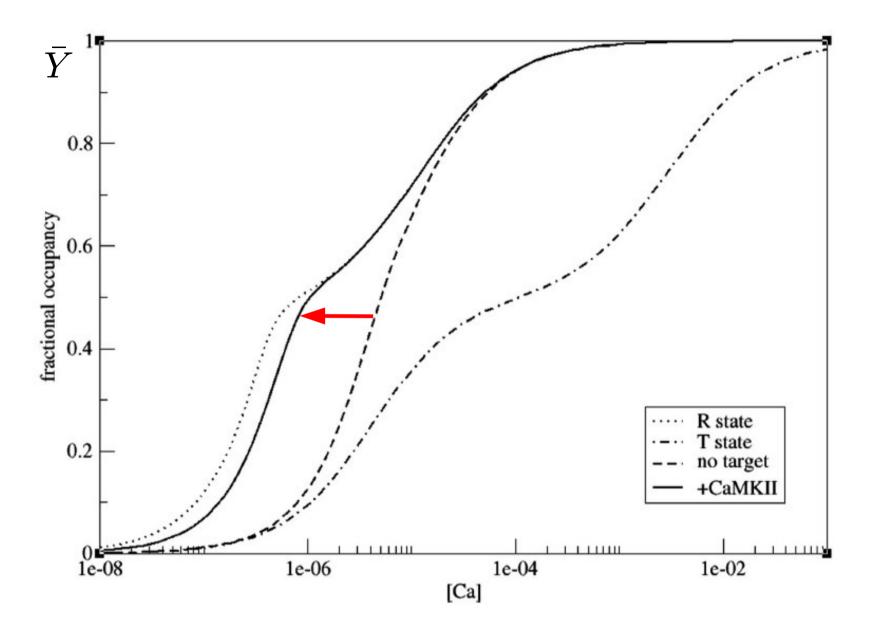
Crouch and Klee (1980) Biochemistry, 19: 3692-3698c

Porumb et al (1994) Anal Biochem 220: 227-237

Peersen et al (1997) Prot Sci 6: 794-807



## Binding to target increases the affinity for Ca<sup>2+</sup>





## **Activity of unsaturated calmodulin**

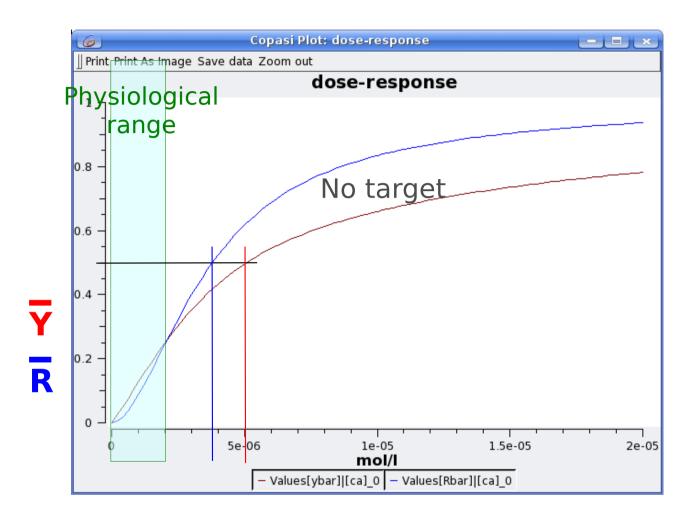
 Fractional activity depends on the number of calcium ions bound. E.g.:

$$\frac{R_2}{T_2} = \frac{1}{L \cdot c^2}$$

- $R_0/T_0 = 1/20000 (1/L)$
- $R_1/T_1 = 1/170$
- $R_2/T_2 = 0.69$   $\longrightarrow$  half-saturation = equi-probability
- $R_3/T_3 = 80$
- $R_4/T_4 = 10000$

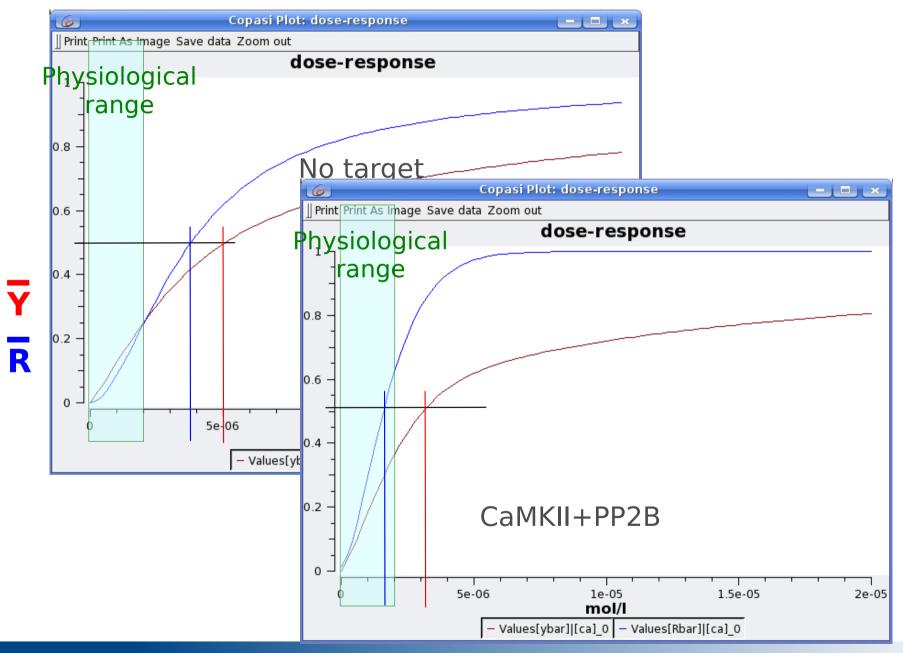


## But ... we're out of the physiological range?



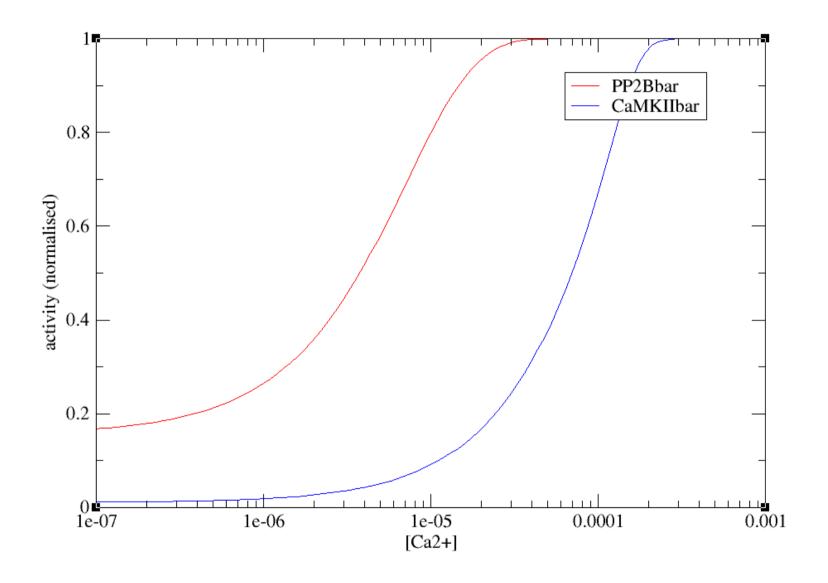


## Targets stabilises Ca<sup>2+</sup> binding: This is systems biology!





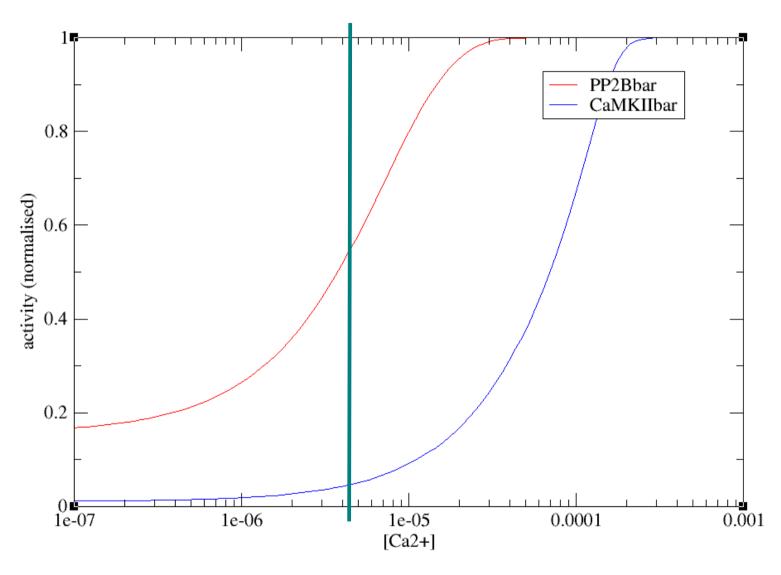
## **Bidirectional synaptic plasticity**





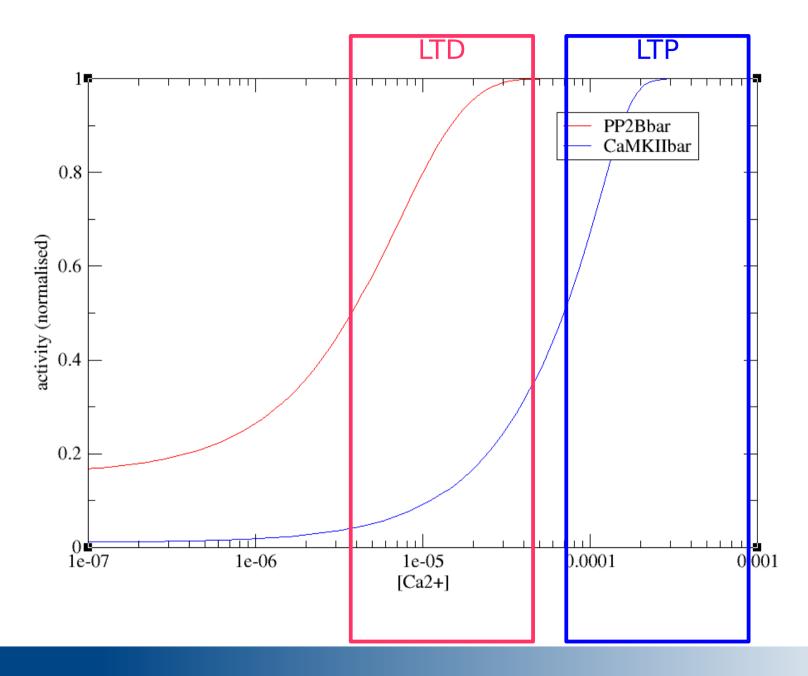
## **Bidirectional synaptic plasticity**

#### half saturation of calmodulin: CaN half activated





## **Bidirectional synaptic plasticity**





## **Conclusions of part 1**

Allosteric model of Calmodulin, with only two states for the EF hands, binding calcium with different affinities, and a concerted transition for all 4 EF hands. Parameters estimated from experimental data-sets.

Model fits independent experimental datasets.

Affinity for calcium increases upon binding of the target.

CaM significantly "active" with less than 4 Ca<sup>2+</sup> bound.

CaM bind its targets with less than 4 Ca<sup>2+</sup> bounds.

The model displays an activation of the sole PP2B at low concentration of calcium, while high concentrations activate both PP2B and CaMKII.



## Wait a minute! Signal transduction is not at equilibrium!

AMPAR post-synaptic potential: 5 ms

Calcium spike: 50 ms

Half saturation calmodulin (kon=1.5e6, koff=100): 5 ms

Relaxation between calmodulin states: 1 ms

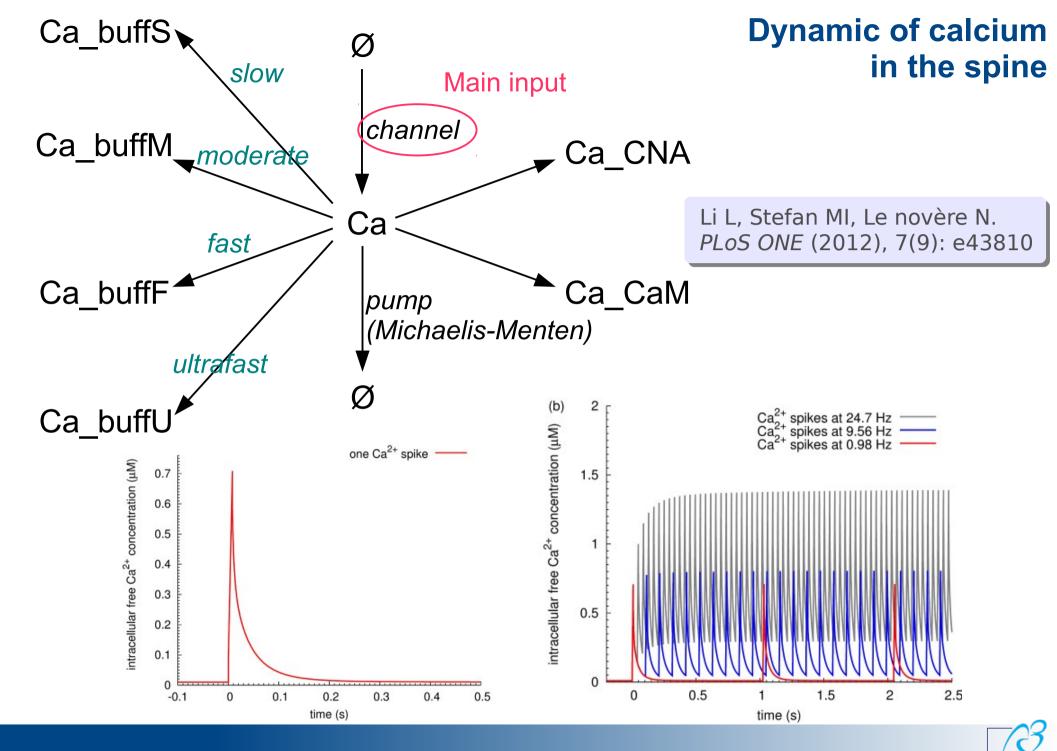
autophosphorylation of CaMKII (kon=6): 100 ms



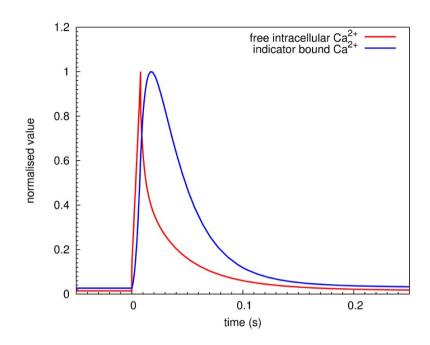


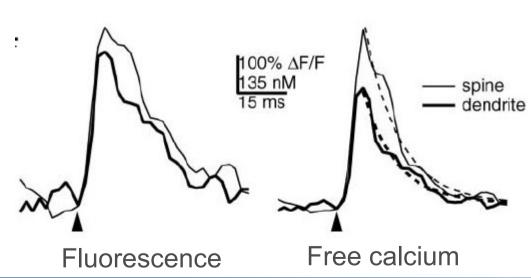


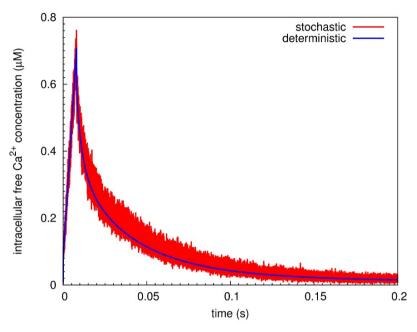




## Are those spikes realistic?





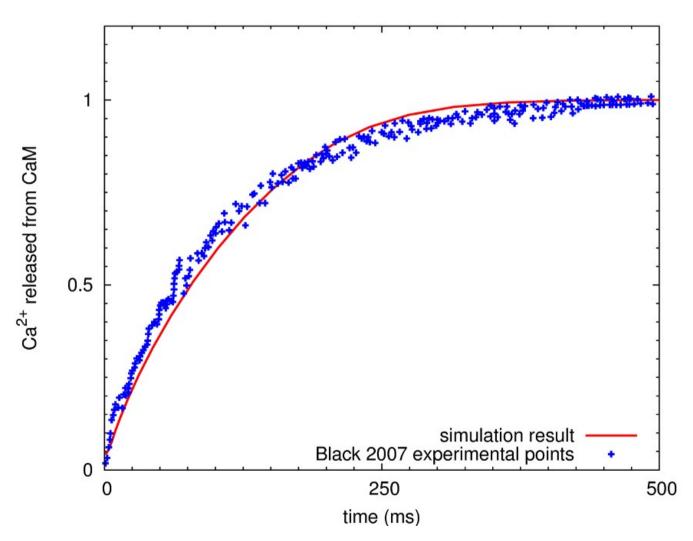


Relative uncertainty increases when concentration decreases, both in concentration and time, but no difference in dynamics.

Sabatini et al (2002) Neuron 33: 439–452.



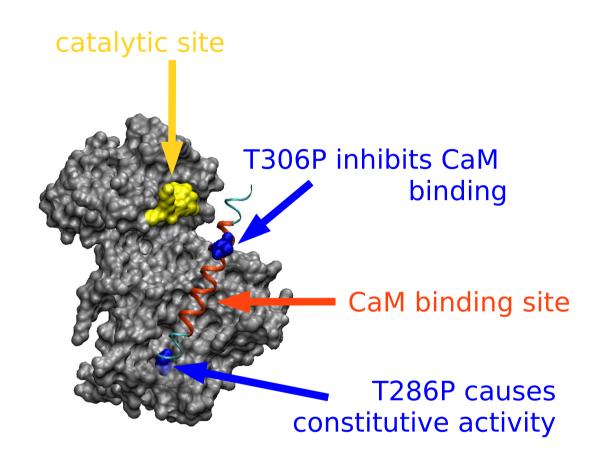
## **Validation of CaM kinetics**



Black DJ, Selfridge JE, Persechini A (2007). Biochemistry 46: 13415-13424.



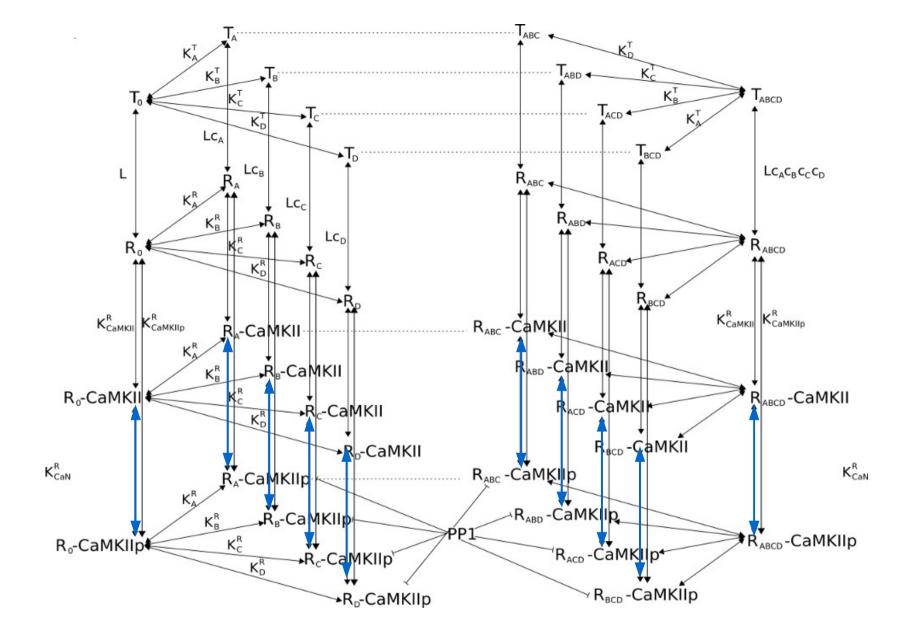
#### Calcium/calmodulin kinase II



Calmodulin trapping is an apparent increase of affinity of CaMKII for CaM when T286 is phosphorylated

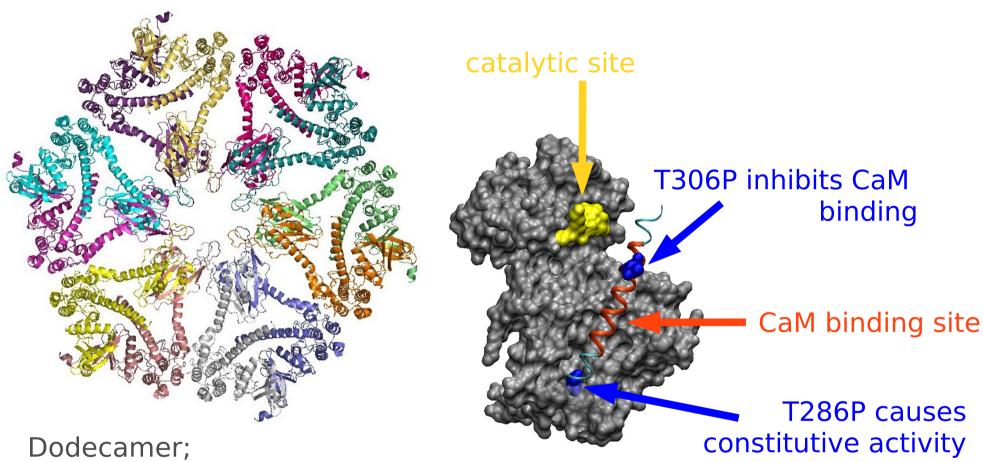
Stefan MI, Marshall D, Le Novère N. PLoS ONE (2012), 7(1): e29406







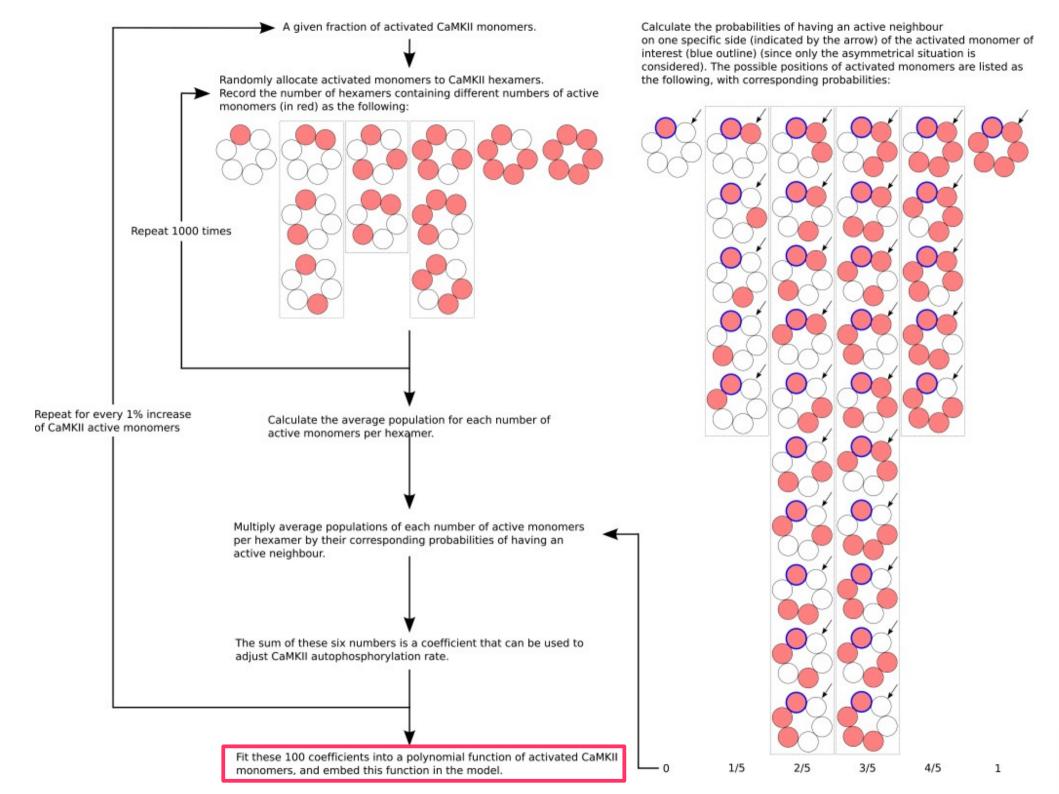
#### Calcium/calmodulin kinase II



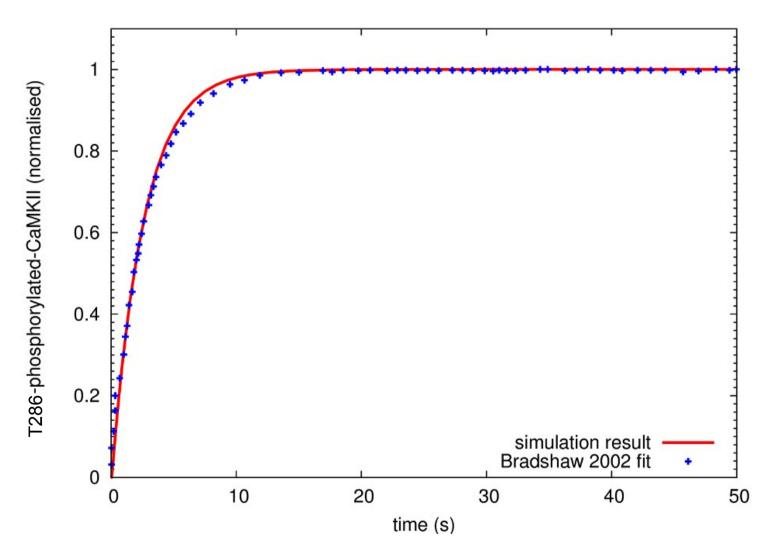
Trans-phosphorylation of T286 by neighbouring subunits

Stefan MI, Marshall D, Le Novère N. PLoS ONE (2012), 7(1): e29406



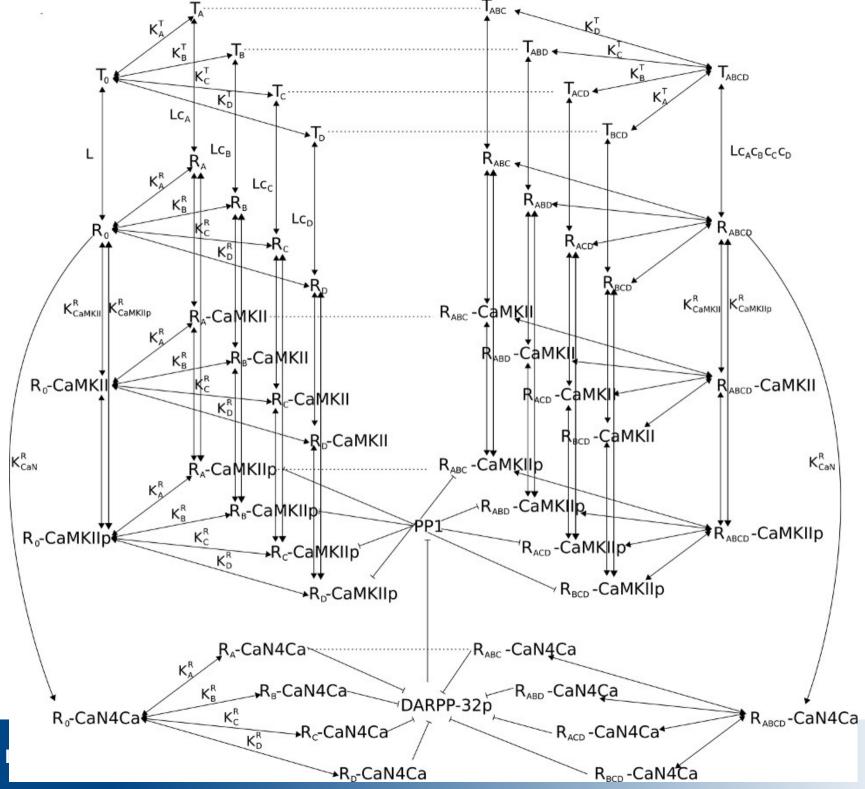


### **Validation of CaMKII kinetics**

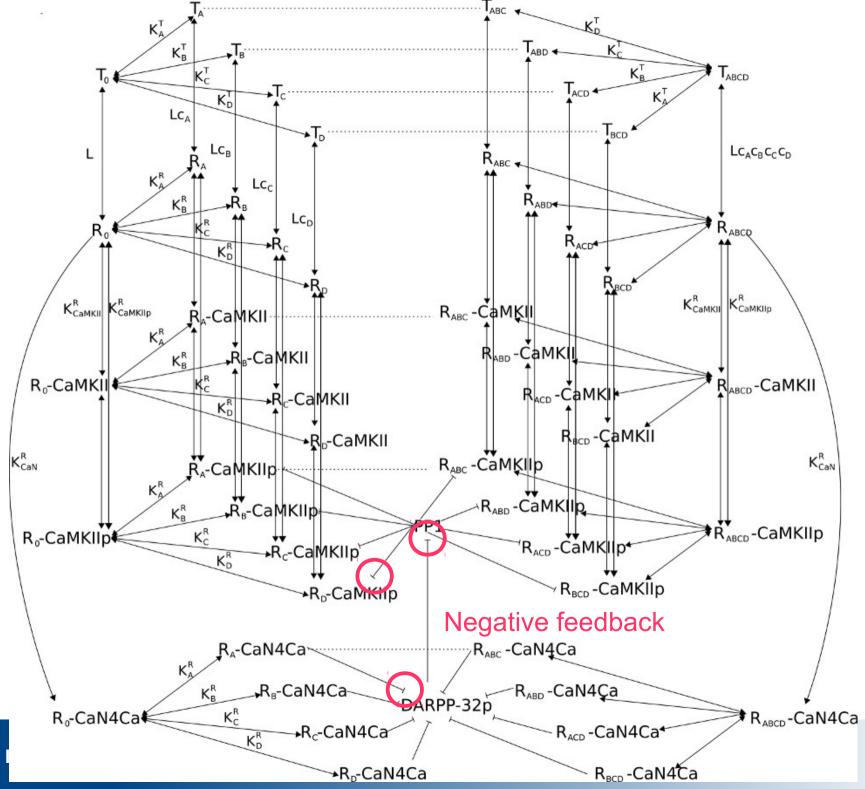


Bradshaw JM, Kubota Y, Meyer T, Schulman H (2003). PNAS 100: 10512-10517.



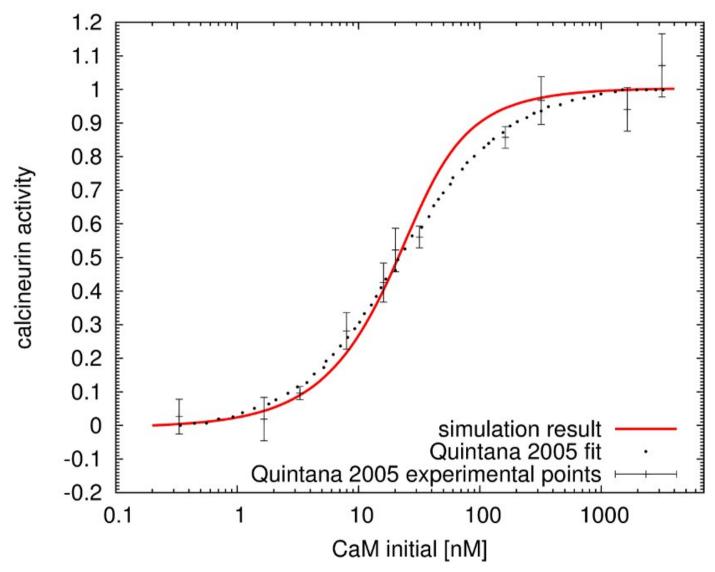






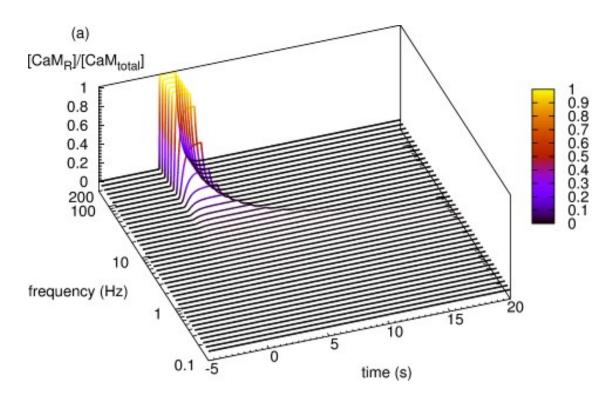


### Validation of calcium-activation of CaN



Quintana AR, Wang D, Forbes JE, Waxham MN (2005). BBRC 334: 674-680.

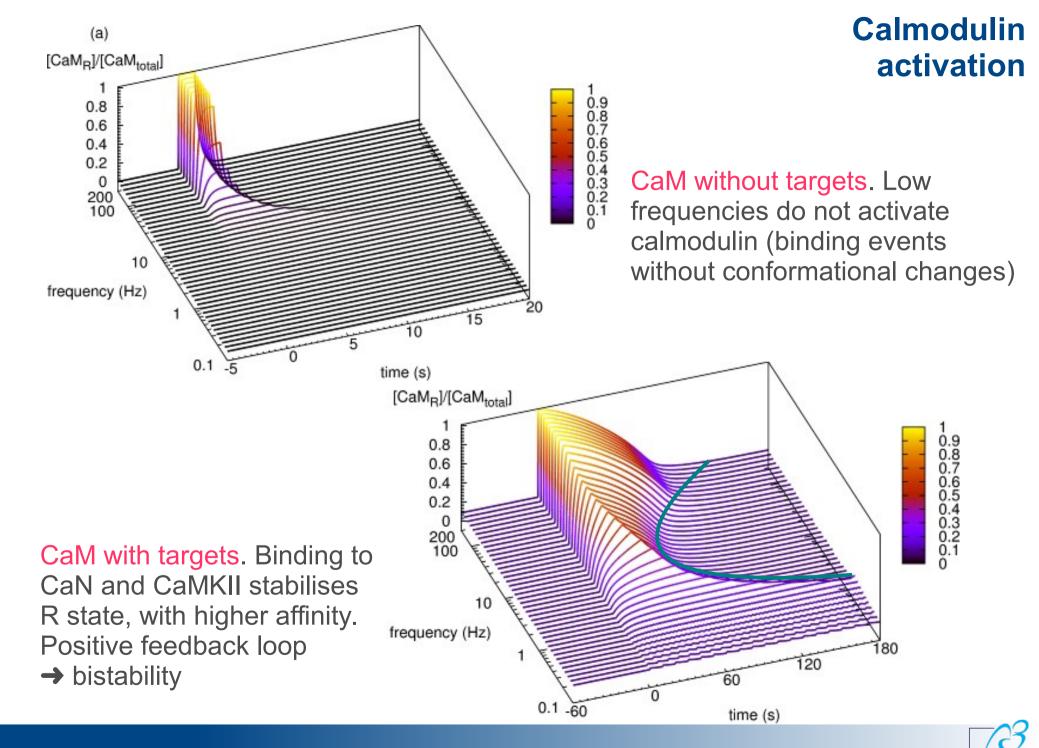


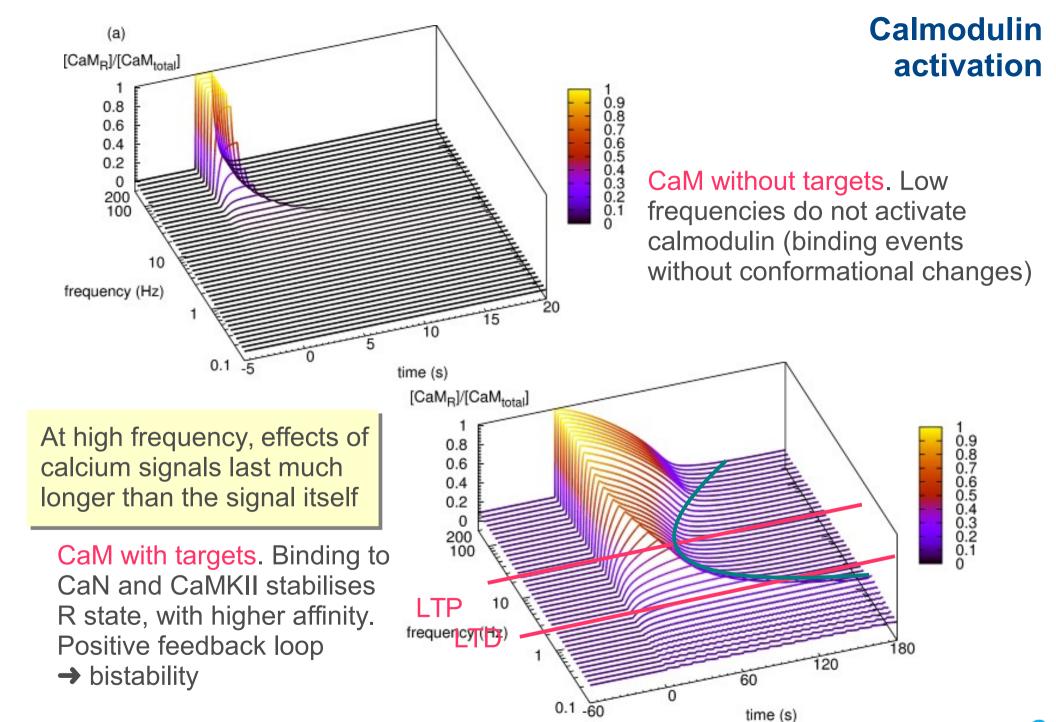


## **Calmodulin** activation

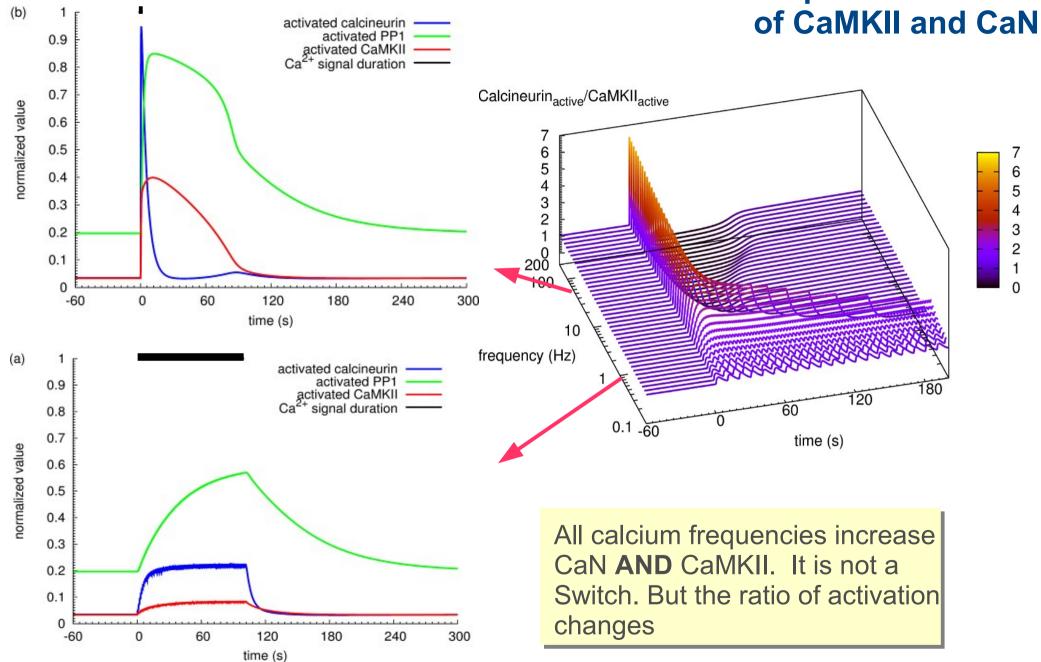
CaM without targets. Low frequencies do not activate calmodulin (binding events without conformational changes)





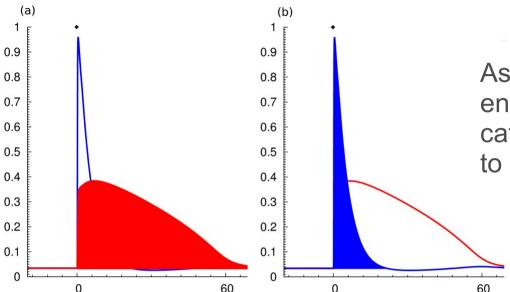


## **Temporal activation**



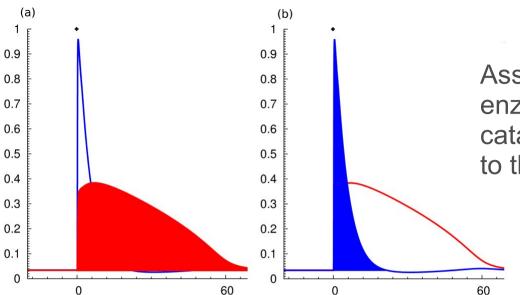


# Bidirectional plasticity



Assuming that catalytic rates of active enzyme do no change, the quantity of catalysed reaction events is proportional to the integral of the activation curve

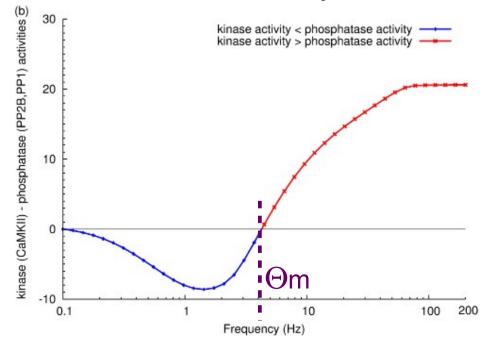




## Bidirectional plasticity

Assuming that catalytic rates of active enzyme do no change, the quantity of catalysed reaction events is proportional to the integral of the activation curve

Bienestock-Cooper-Munro (BCM) curve: difference of active areas\*catalytic activities





#### (a) (b) 0.9 0.9 0.8 8.0 0.7 0.7 0.6 0.6 0.5 0.5 0.4 0.4 0.3 0.3 0.2 0.2

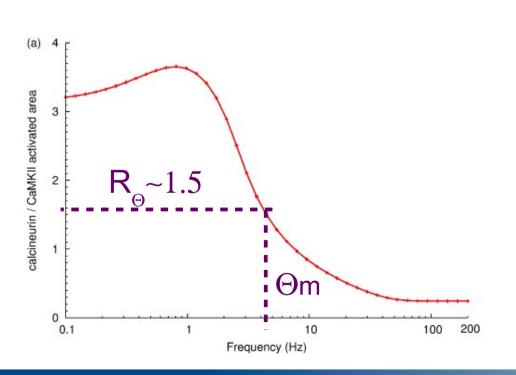
0.1

60

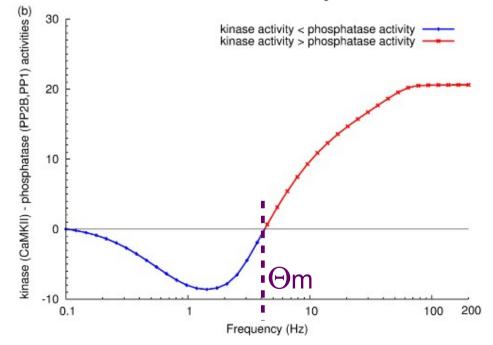
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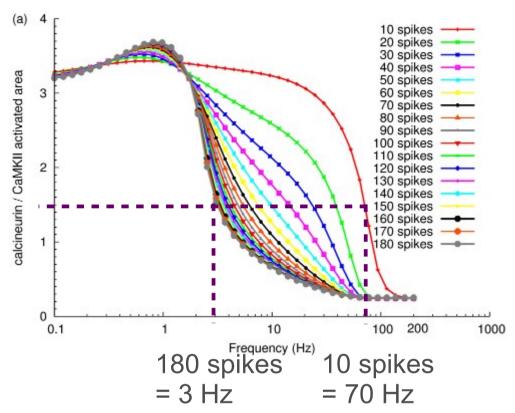


0.1

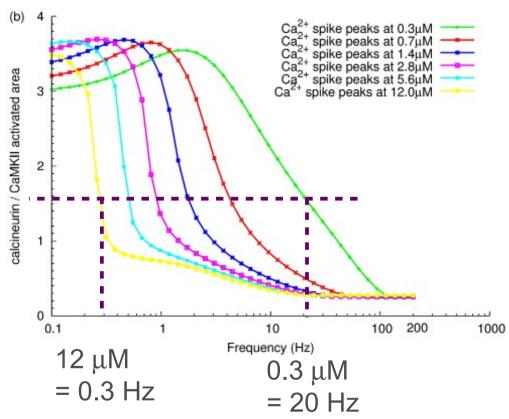
0

0

#### Effect of calcium duration and amount

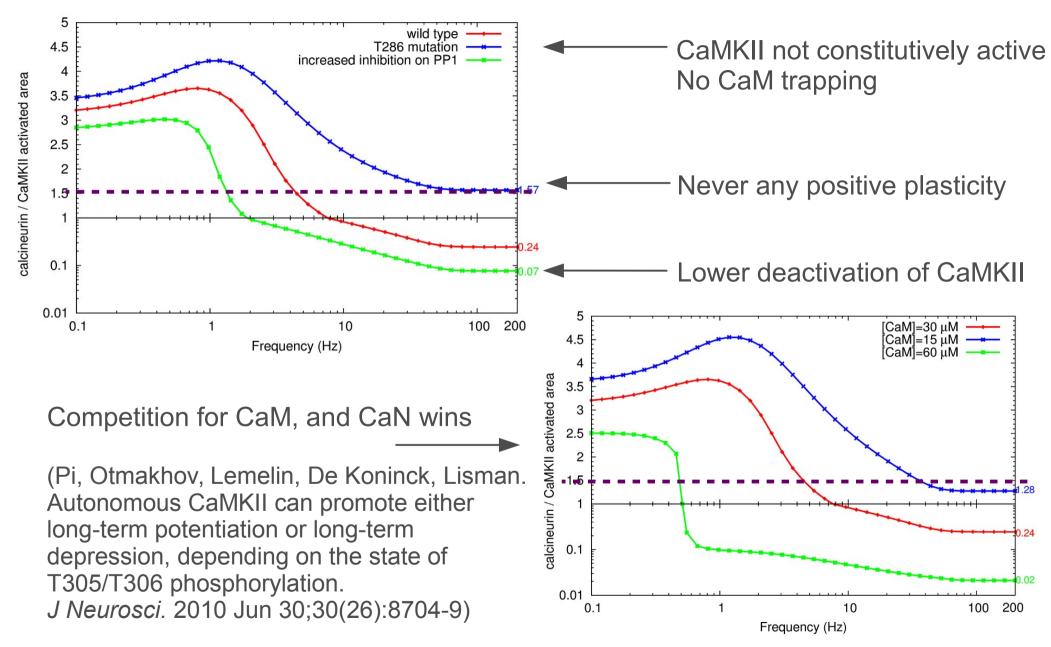


Prolonged or intense signals
Decrease the threshold
frequency: It is not an intrinsic
property of the synapse





### **Effect of intrinsic system perturbations**





## **Summary of part 2**

Allosteric stabilisation by targets triggers bistable CaM response to calcium. Above a certain frequency, CaM activation lasts longer than the initial signal.

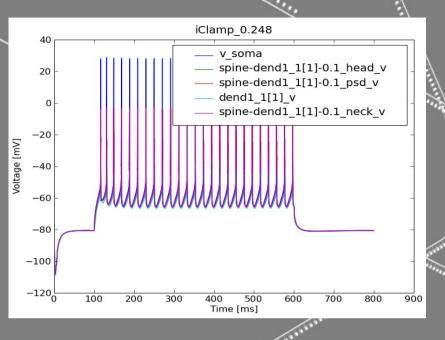
Calcium signals do not choose between CaN and CaMKII, BOTH enzymes are activated at ALL frequencies. The ratio of activity changes.

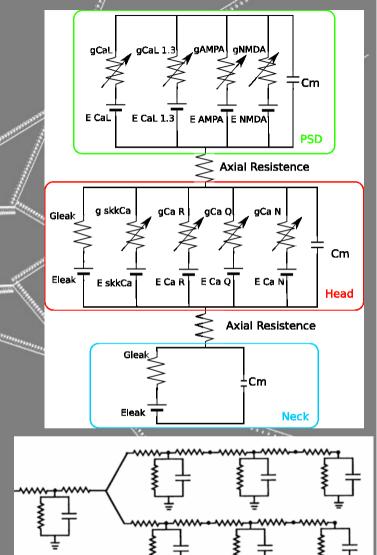
The frequency at which a synapse switches from a depression to a potentiation mode is not an intrinsic property of the synapse, but depends on the length and amplitude of stimulations.

Modifications of topology (T286A), parameters (PP1 inhibition) and initial conditions ([CaM]) affect both response intensity and threshold frequency. Some mutants can't have positive plasticity for any stimulation.

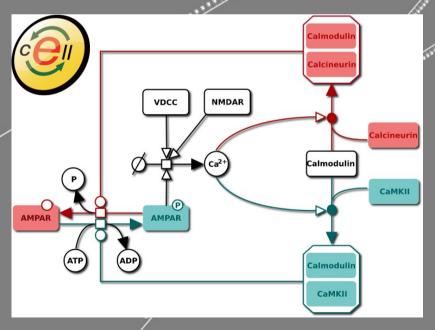


## Whole cell electro-biochemical models

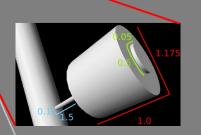




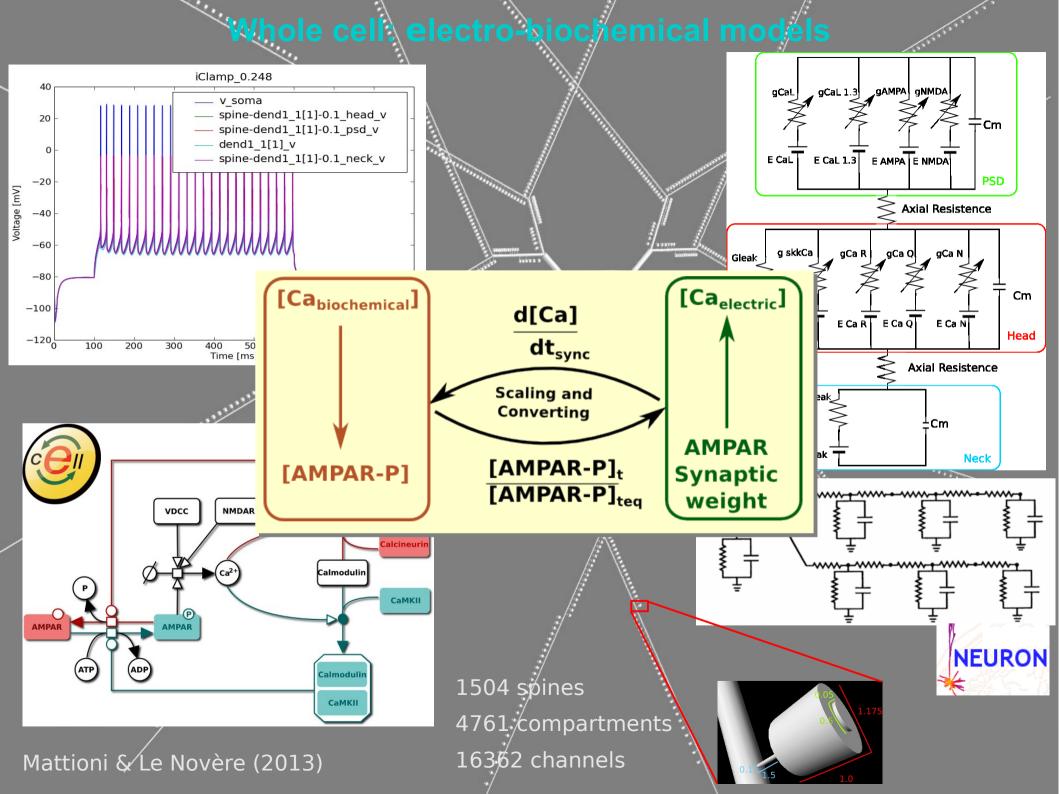
**NEURON** 



1504 spines 4761 compartments 16362 channels



Mattioni & Le Novère (2013)



- Developers of ECell3
- **Developers of COPASI**
- Developers of Scilab
- Developers of Gnuplot



Melanie Stefan



Lu Li







Stuart Edelstein

















## **Extended MWC model necessary for Calmodulin**

$$\bar{Y} = \frac{1}{n} \frac{\sum_{i} \left(\alpha_{i} \prod_{j \neq i} (1 + \alpha_{j})\right) + L \prod_{k} \left(\frac{1 + e_{k} \gamma_{k}}{1 + \gamma_{k}}\right) \sum_{i} \left(c_{i} \alpha_{i} \prod_{j \neq i} (1 + c_{j} \alpha_{j})\right)}{\prod_{i} (1 + \alpha_{i}) + L \prod_{k} \left(\frac{1 + e_{k} \gamma_{k}}{1 + \gamma_{k}}\right) \prod_{i} (1 + c_{i} \alpha_{i})}$$

- 1) Any number of different sites per protomer
- 2) Several protomers can be carried by one subunit

Based on Rubin and Changeux (1966) *J Mol Biol*, 21: 265-274

- $\alpha i = [\text{ligand}]/K^{\text{R}}_{i,\text{lig}}$
- $\gamma k = [\text{modulator}]/K^{R}_{k,\text{mod}}$
- $Ci = K_{i,lig}^R / K_{i,lig}^T$
- $ek = K_{k,mod}^R / K_{k,mod}^T$

Stefan M.I., Edelstein S.J., Le Novère N. BMC Systems Biology (2009), 3: 68



## Simplification of the model for finding *L* and *c*

- Hypothesis for the whole model: free energy of conformational transition is evenly distributed: c is unique
- Additional simplification to determine L: affinities are identical

$$\bar{Y} = \frac{\alpha(1+\alpha)^3 + L\left(\frac{1+\gamma e}{1+\gamma}\right)c\alpha(1+c\alpha)^3}{(1+\alpha)^4 + L\left(\frac{1+\gamma e}{1+\gamma}\right)(1+c\alpha)^4}$$



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- Model constraints for the determination of c and L
  - Ca binding in presence of target: none, skMLCK, PhK5,
     CaATPase (Peersen et al (1997) Prot Sci 6: 794-807).
     Concentration at 50% saturation.
  - 100 000 parameter sets plus least-square
  - 13 identical minima. e for skMLCK is 10<sup>-15</sup>, which can be taken as skMLCK binding only to R state.



## Relaxation of the model for finding individual Kd

Determination of individual affinities:

$$\bar{Y} = 0.25 \frac{\sum_{i} \left(\alpha_{i} \prod_{j} (1 + \alpha_{j})\right) + L \sum_{i} \left(c \alpha_{i} \prod_{j} (1 + c \alpha_{j})\right)}{\prod_{i} (1 + \alpha_{i}) + L \prod_{i} (1 + c \alpha_{i})}$$



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- Model constraints for calcium dissociation constants
  - Complete CaM (Bayley et al (1996) Prot Sci 5: 1215-1228)
  - N and C term Mutants (Shifman et al (2006) PNAS, 103: 13968-13973)
  - R-only skMLCK (Peersen et al (1997) Prot Sci 6: 794-807)
  - Concentration at 25% and 50% saturation.
  - Systematic logarithmic sampling of the affinity space (coarse-grained, 50 values per affinity, then refined 66 values per affinity) = 25 millions parameter sets

